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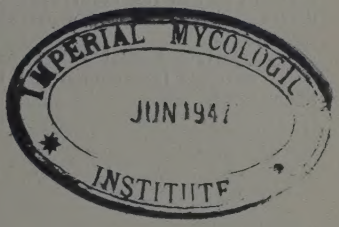


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*MATHÉMATIQUES*

# SUR LES LIGNES DE NIVEAU DES FONCTIONS CONTINUES POSSÉDANT DES DÉRIVÉES PARTIELLES

Par G. M. ADELSON-WELSKY et A. S. KRONRODE

*(Présenté par N. N. Lusin, de l'Académie, le 16. V. 1945)*

Définition 1. On appelle ensemble de niveau $E_{u_0}$ d'une fonction réelle $u(x, y)$ l'ensemble des points $(x, y)$ tels que $u(x, y) = u_0$.

Si $u(x, y)$ est une fonction continue, l'ensemble $E_{u_0}$ est fermé.

On suppose dans cette Note que

A. $G$ est un domaine plan simplement connexe dont la frontière $C$ est homéomorphe à une circonférence.

B. $u(x, y)$ est une fonction réelle continue, dont $M$ et $m$ sont le maximum et le minimum, définie dans un domaine $G$ et possédant en chaque point de $G$ les deux dérivées partielles du premier ordre.

Il suit évidemment du théorème de Vitali que l'on a le

Lemme 1. Soit $u(x, y)$ une fonction arbitraire. Alors pour chaque $y_0$ (resp. $x_0$) l'ensemble $W_{y_0}$ (resp. $W_{x_0}$) des valeurs de $w$ telles qu'il existe un point $x, y_0$ (resp. $x_0, y$) pour lequel $u(x, y_0) = w$ (resp. $u(x_0, y) = w$) et $\partial u(x, y_0)/\partial x = 0$ (resp. $\partial u(x_0, y)/\partial y = 0$) est de mesure nulle.

Théorème 1. *Soit $u(x, y)$ une fonction vérifiant les conditions* A *et* B. *Alors le segment $mM$ contient un ensemble $\tilde{\mathfrak{U}}$ de mesure $M - m$ et tel que pour chaque $E_u (u \in \tilde{\mathfrak{U}})$:*

1°. *L'ensemble $I_1$ des points où l'on a $\partial u/\partial y = 0$ a pour projection sur l'axe $x$ un ensemble de mesure nulle.*

2°. *L'ensemble $I_2$ des points où l'on a $\partial u/\partial x = 0$ a pour projection sur l'axe $y$ un ensemble de mesure nulle.*

Démonstration. Considérons un système de droites $l_c (x = c)$. Soit $W_c$ l'ensemble des niveaux $w$ pour lesquels le produit $E_w \frown l_c$ contient un point où $\partial u/\partial y = 0$. En vertu du lemme 1 l'ensemble $W_c$ est de mesure nulle quel que soit $c$.

Soit $P$ l'ensemble des points $(x_0, u_0)$ du plan $xu$ tel que $E_{u_0} \frown l_{x_0}$ contient un point où $\partial u/\partial y = 0$. En vertu du théorème de Fubini le segment $mM$ contient un ensemble $\mathfrak{U}_1$ de mesure $M - m$ tel que chaque droite $u = u_0$ coupe $P$ en un ensemble de mesure nulle si $u_0 \in \mathfrak{U}_1$. En échangeant les rôles de $x$ et $y$ dans nos raisonnements, nous obtenons un ensemble $\mathfrak{U}_2$. L'ensemble $\tilde{\mathfrak{U}} = \mathfrak{U}_1 \frown \mathfrak{U}_2$ vérifie les conditions du théorème.

Théorème 2. *Soit $u(x, y)$ une fonction vérifiant les conditions* A *et* B *et $\tilde{\mathfrak{U}}$ — l'ensemble des niveaux dont l'existence est démontrée par le théorème* 1. *Alors pour chaque $u_0 \in \tilde{\mathfrak{U}}$ chaque composante de l'ensemble $E_{u_0}$ est localement connexe.*

Démonstration. Soit $K$ une certaine composante de $E_{u_0}$ et $a$ un point où $K$ n'est pas localement connexe. Alors il existe un cercle $R_a$ de centre $a$

et dont la frontière est $C_a$ tel que dans le cercle $R_a$ le continu $K$ contient une suite de composantes $K_i$ dont chacune possède un point $b_i$ sur $C_a$ et d'ailleurs le point $a$ appartient à la limite inférieure topologique de la suite $\{K_i\}$. Sans restreindre la généralité, on peut supposer que les points $b_i$ convergent vers un point $b \subset C_a$.

Supposons pour fixer les idées que les coordonnées $x$ des points $a$ et $b$ sont différentes et respectivement égales à $x_1$ et $x_2 (x_1 < x_2)$. Alors chaque droite $l_{x_0} (x = x_0,\ x_1 < x_0 < x_2)$ coupe une infinité dénombrable de $K_i$. Mais $K_i \frown K_j = 0$, donc sur chaque droite $l_{x_0}$ il existe un point $(x_0, t_{x_0})$, limite pour les points de $K_i \frown l_{x_0}$, donc $\partial u\,(x_0, t_{x_0})/\partial y = 0$. Cela contredit à 1° du théorème 1. Donc le théorème 2 est démontré.

Définition 2. On appelle point de ramification d'un continu $K$ localement connexe un point $a$ au voisinage duquel $K$ contient trois arcs simples $l_1$, $l_2$ et $l_3$ tels que $l_1 \frown l_2 = l_2 \frown l_3 = l_1 \frown l_3 = a$.

Lemme 2. Un continu plan $K$ borné et localement connexe qui n'est ni un arc simple ni une courbe homéomorphe à une circonférence contient des points de ramification.

Démonstration. 1°. Supposons que $K$ contienne une partie $L$ homéomorphe à une circonférence, $L \neq K$. Un point $a \subset K - L$ peut être joint avec un point arbitraire $b \subset L$ au moyen d'un arc simple $l$. Considérons le premier (si l'on compte de $a$ vers $b$) point d'intersection $l \cap L$. Ce point est un point de ramification.

2°. Supposons que $K$ ne contient pas de partie homéomorphe à une circonférence.

Soient $a \subset K$, $b \subset K$ et $c \subset K$. Il existe des arcs simples $l_a \subset K$, $l_b \subset K$ et $l_c \subset K$ dont les extrémités sont respectivement $b$ et $c$, $c$ et $a$, $a$ et $b$. Si aucun des trois arcs $l_a$, $l_b$ et $l_c$ ne contient la somme des deux autres, alors $l_a + l_b + l_c$ contient ou bien une courbe homéomorphe à une circonférence, ou bien un point de ramification. Et si de quelque manière que l'on prend les trois points $a$, $b$ et $c$ de $K$ cette circonstance n'a pas lieu, l'ensemble $K$ est ordonné et $K$ est un arc simple.

Théorème 3 (fondamental). *Soit $u(x, y)$ une fonction vérifiant les conditions* A *et* B. *Alors il existe sur le segment $mM$ un ensemble $\mathfrak{U}$ de mesure $M - m$ tel que pour chaque $u \subset \mathfrak{U}$ les ensembles de niveau $E_u$ ne contiennent que les composantes suivantes*: a) *des courbes homéomorphes aux circonférences*, b) *des arcs simples dont les extrémités se trouvent sur $C$*, c) *des points, et d'ailleurs pour chaque $u \subset \mathfrak{U}$ l'ensemble de points qui sont des composantes a pour projections sur les arcs $x$ et $y$ des ensembles de mesure nulle.*

Démonstration. En vertu du théorème 2, on peut extraire l'ensemble $\tilde{\mathfrak{U}}$ des niveaux pour lesquels chaque composante $K \subset E_{u_0}$ pour $u_0 \subset \tilde{\mathfrak{U}}$ est localement connexe. En vertu d'un théorème de Komarevsky [1] un système de continus plans n'empiétant pas et contenant des points de ramification est au plus dénombrable. Donc, en vertu du lemme 2, il existe au plus une infinité dénombrable de composantes qui ne sont ni points, ni arcs simples, ni homéomorphes à des circonférences. Enlevons de $\tilde{\mathfrak{U}}$ tous les $u$ pour lesquels $E_u$ contient de telles composantes exceptionnelles. Soit $\mathfrak{U}$ l'ensemble qui reste.

Soit $a(x_0, y_0) \subset E_{u_0}$, $(u_0 \in \mathfrak{U})$ et $\partial u\,(x_0, y_0)/\partial x \neq 0$ ou bien $\partial u\,(x_0, y_0)/\partial y \neq 0$. Pour fixer les idées supposons que $\partial u\,(x_0, y_0)/\partial y > 0$.

Choisissons un segment $L$ à extrémités $\beta(x_0, y_0 - \varepsilon)$ et $\gamma(x_0, y_0 + \varepsilon)$ de telle manière que pour $0 < \delta \leqslant \varepsilon$ on ait $u(x_0, y_0 + \delta) > u(x_0, y_0) = u_0$ et $u(x_0, y_0 - \delta) < u(x_0, y_1) = u_0$. Les points $\beta$ et $\gamma$ sont séparés par l'ensemble $E_{u_0}$, donc, en vertu d'un théorème topologique connu [2], ils sont séparés par sa composante $K$, donc elle n'est pas un point. $K \subset E_{u_0}$ coupe le segment $L$, donc il contient le point $a$. Si $K$ n'est pas homéomorphe à une circonférence, $K$ est un arc simple, et comme $K$ divise $G$, les extrémités de $K$ sont sur la frontière $C$ de $G$. Comme pour $u \in \mathfrak{U}$ chaque composante de $K \subset E_u$ qui n'est pas

un point doit en vertu du théorème 1 contenir un point où $\partial u/\partial x \neq 0$, ou bien $\partial u/\partial y \neq 0$, l'ensemble de composantes des ensembles de niveau $E_u (u \in \mathfrak{U})$ ne contient que des points, des parties homéomorphes à une circonférence et des arcs simples dont les extrémités sont sur $C$. Dans les points qui sont des composantes les deux dérivées partielles s'annulent, donc pour chaque niveau $u_0 \in \mathfrak{U} \subset \tilde{\mathfrak{U}}$ les projections de ces points forment, en vertu du théorème 1, un ensemble de mesure nulle.

Le théorème 3 est démontré.

Supposons maintenant que la fonction $u(x, y)$ vérifie les conditions A et B et possède d'ailleurs une différentielle totale en tous les points de $G$.

Alors, en vertu d'un théorème de Young [3] en tous les points $(x, y)$ où $(\partial u/\partial x)^2 + (\partial u/\partial y)^2 \neq 0$ l'ensemble de niveau $E_{u(x,y)}$ possède une tangente. Il en résulte le

Théorème 4. *Soit $u(x, y)$ une fonction vérifiant les conditions* A *et* B *et possédant une différentielle totale. Alors sur le segment $mM$ il existe un ensemble $\mathfrak{U}$ de mesure $M - m$ tel que pour $u \in \mathfrak{U}$ les ensembles de niveau $E_u$ ne contiennent que les composantes suivantes*: a) *des courbes homéomorphes à une circonférence*, b) *des arcs simples ayant leurs extrémités sur $C$*, c) *des points, et d'ailleurs pour chaque niveau $u \subset \mathfrak{U}$ l'ensemble $I_u$, formé des points de $E_u$ qui sont des composantes et des points où $E_u$ n'a pas de tangente, a pour projections sur les axes $x$ et $y$ des ensembles de mesure nulle.*

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Moscou.

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LITTÉRATURE CITÉE

[1] W. Komarevsky, Trav. de l'Univ. d'Etat à Turkestan, fasc. 6 et 8, p. 21 (1923). [2] F. Hausdorff, Grundzüge der Mengenlehre, p. 343 (1914). [3] Ch. de la Vallée-Poussin, Cours d'analyse, t. 1, p. 161, 1933.

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*MATHÉMATIQUES*

# SUR LA VALEUR ASYMPTOTIQUE DE LA MEILLEURE APPROXIMATION D'UNE FONCTION QUI POSSÈDE UN POINT CRITIQUE RÉEL

Par I. I. IBRAGUIMOFF

*(Présenté par S. N. Bernstein, de l'Académie, le 10. VI. 1945)*

M. Serge Bernstein indique dans son article(1) une méthode pour calculer la valeur asymptotique de la meilleure approximation de la fonction $f(x) = (a-x)^s$ où $a > 1$ et $s$ est un nombre réel donné arbitraire.

En modifiant légèrement cette méthode de la Vallée-Poussin(2) l'a appliquée au calcul de la valeur asymptotique de la meilleure approximation de la fonction

$$f(\cos\vartheta) = (\operatorname{ch} a - \cos\vartheta)^s [\lg(\operatorname{ch} a - \cos\vartheta)]^m$$

au moyen d'un polynôme trigonométrique.

J'applique la méthode de M. Bernstein au calcul de la valeur asymptotique de la meilleure approximation de la fonction de la forme

$$f(x) = (a-x)^s [\lg(a-x)]^m, \quad a > 1 \tag{1}$$

sur le segment $[-1, +1]$ au moyen d'un polynôme de degré $n$.

On prend pour base des recherches le fait suivant. Si un polynôme $P_n(x)$ de degré $n$ et une fonction $f(x)$ coïncident aux points $x_1, x_2, \ldots, x_{n+1}$ du segment $[-1, +1]$ où le polynôme

$$R(x) = (x-a)\cos(n\vartheta + \delta) =$$

$$= \frac{1}{2}\{(x + \sqrt{x^2-1})^n [ax - 1 + \sqrt{(a^2-1)(x^2-1)}] +$$

$$+ (x - \sqrt{x^2-1})^n [ax - 1 - \sqrt{(a^2-1)(x^2-1)}]\} \tag{2}$$

de degré $n+1$ s'annule, on a la formule suivante

$$F(x) = f(x) - P_n(x) = \frac{R(x)}{2\pi i} \int_C \frac{f(z)\,dz}{(z-x)R(z)} \tag{3}$$

où $C$ est un contour quelconque entourant le segment $(-1, +1]$, à l'intérieur duquel $f(z)$ est régulière. Pour la fonction $f(z)$, définie par la formule (1), le contour $C$ se déforme en une circonférence dont le rayon est infini et qui possède une coupure infiniment petite sur l'axe positif; des deux extrémités de cette coupure le long de l'axe réel il passe deux droites infiniment proches qui se terminent par une circonférence de rayon infiniment petit et de centre $a$.

1. Supposons que $s$ est un nombre réel donné positif (non entier) et d'ailleurs arbitraire, $m$ un nombre positif donné. Dans ce cas dans la for-

mule (3) les intégrales le long des deux circonférences sont nulles et l'on obtient ainsi les formules

$$F(x)=(a-x)^s[\lg(a-x)]^m-P_n(x)=$$

$$=\frac{R(x)}{2\pi i}\int_a^{\infty}\frac{(z-a)^s\{e^{i\pi s}[\lg(z-a)+i\pi]^m-e^{-i\pi s}[\lg(z-a)-i\pi]^m\}\,dz}{(z-x)\,R(z)} \tag{4}$$

d'où il résulte que l'on a l'égalité asymptotique

$$F(x)\sim\frac{(-1)^m\sin\pi s\,(a^2-1)^{\frac{s-1}{2}}(\lg n)^m\,\Gamma(s+1)}{\pi n^{s+1}(a+\sqrt{a^2-1})^n}\cos(n\vartheta+\delta) \tag{5}$$

et nous arrivons à la conclusion suivante.

Théorème 1. *Si $s$ est un nombre réel (non entier) positif et $m$ est un nombre positif quelconque, la meilleure approximation asymptotique de la fonction $f(x)=(a-x)^s[\lg(a-x)]^m$, $a>1$, sur le segment $[-1,+1]$ au moyen d'un polynôme de degré $n$ est définie par la formule*

$$E_n\{(a-x)^s[\lg(a-x)]^m\}\sim\frac{\sin\pi s(a^2-1)^{\frac{s-1}{2}}(\lg n)^m\,\Gamma(s+1)}{\pi n^{1+s}(a+\sqrt{a^2-1})^n} \tag{I}$$

En particulier pour $m=0$ on obtient de la formule (1) la formule connue de M. Bernstein (3).

2. Si $s=p$ est un entier positif et $m$ un nombre positif arbitraire, alors, en tenant compte de la régularité de la fonction $(a-x)^p$ au point $a$ on obtient d'une manière analogue le

Théorème 2. *Si $p$ est un entier positif et $m$ un nombre positif arbitraire, la meilleure approximation asymptotique de la fonction $f(x)=(a-x)^p[\lg(a-x)]^m$ sur le segment $[-1,+1]$ au moyen d'un polynôme de degré $n$ est définie par la formule*

$$E_n\{(a-x)^s[\lg(a-x)]^m\}\sim\frac{m\sin\pi s\,(a^2-1)^{\frac{p-1}{2}}(\lg n)^{m-1}\,\Gamma(p+1)}{n^{p+1}(a+\sqrt{a^2-1})^n} \tag{II}$$

Le cas particulier où $p=m=1$ a été considéré par M. Fernstein (3).

3. En supposant que $s$ est un nombre réel (non entier) positif arbitraire et en faisant $a\,(a>1)$ tendre vers 1, on obtient de la formule (4)

$$F(x)=-\frac{\sin\pi s}{\pi}(1-x)\,T_n(x)\int_1^{\infty}\frac{(z-1)^{s-1}[\lg(z-1)]^m\,dz}{(z-x)\,T_n(x)} \tag{6}$$

Dans ce cas nous ne pouvons pas dire que $P_n(x)$ est le polynôme qui s'écarte le moins de $(1-x)^s[\lg(1-x)]^m$ parce que la différence $F(x)=(1-x)^s[\lg(1-x)]^m-P_n(x)$ possède au plus $n+1$ extrêmes absolus à signes alternés; cependant nous trouvons une borne supérieure et une borne inférieure pour la meilleure approximation de la fonction donnée.

En posant d'abord $z+\sqrt{z^2+1}=\sigma$ et ensuite $\sigma=1+u/n$ pourvu que $|x|<1$, on obtient de l'égalité (6)

$$F(x)=(-1)^{m+1}\frac{\sin\pi s2^{m-s+2}[\lg\sqrt{2}\,n]^m\,T_n(x)}{\pi n^{2s}}\int_0^{\infty}\frac{u^{2s-1}\,du}{(e^u+e^{-u})\left(1+\frac{u^2}{2n^2(1-x)}\right)}$$

Donc, si $n^2(1-x)=b\to\infty$ on a

$$F(x)=\frac{(-1)^{m+1}\sin\pi s2^{m-s+2}}{\pi n^{2s}}[\lg\sqrt{2}\,n]^m\,T_n(x)\int_0^{\infty}\frac{u^{2s-1}\,du}{e^u+e^{-u}}+\frac{\lg n\,\varepsilon_n}{n^{2s}\lg n} \tag{7}$$

et, en général, pour chaque $b \geqslant 0$, nous avons

$$F(x) = (-1)^{m+1} \frac{\sin \pi s 2^{m-s+2}}{\pi n^{2s}} [\lg \sqrt{2}\, n]^m T_n(x) \int_0^\infty \frac{b u^{2s-1}\, du}{(e^u + e^{-u}) \left(b + \frac{u^2}{2}\right)} + \frac{\varepsilon_n \lg n}{n^{2s}} \qquad (8)$$

Il résulte des inégalités (7) et (8) que l'on a le

Théorème 3. *Si $s$ est un nombre positif (non entier) quelconque et $m$ un nombre positif arbitraire, la meilleure approximation asymptotique de la fonction $(1-x)^s [\lg (1-x)]^m$ vérifie l'inégalité*

$$C_s > \frac{n^{2s}}{(\lg \sqrt{2}\, n)^m} E_n \{(1-x)^s [\lg (1-x)]^m\} > \left(\frac{\pi}{\pi+4}\right)^{2s} \frac{C_s}{2\sqrt{2}} \qquad (III)$$

*où*

$$C_s = \frac{2^{m-s+2} \sin \pi s}{\pi} \int_0^\infty \frac{u^{2s-1}\, du}{e^u + e^{-u}}$$

Le cas $m = 0$ a été considéré par M. Bernstein.

4. En supposant que $s = p$ est un entier positif et en faisant $a$ $(a > 1)$ tendre vers l'unité, on obtient de (5) dans la condition $|x| < 1$

$$F(x) = (-1)^m \frac{m 2^{m-p+1} [\lg \sqrt{2}\, n]^{m-1}}{n^{2p}} T_n(x) \int_0^\infty \frac{u^{2p-1}\, du}{(e^u + e^{-u}) \left(1 + \frac{u^2}{2(1-x)\, n^2}\right)} + \frac{\varepsilon_n}{n^{2p}} \qquad (9)$$

ou bien pour $n^2 (1-x) = b \to \infty$,

$$F(x) = (-1)^m \frac{m 2^{m-p+1} [\lg \sqrt{2}\, n]^{m-1} T_n(x)}{n^{2p}} \int_0^\infty \frac{u^{2p-1}\, du}{e^u + e^{-u}} + \frac{\varepsilon_n \lg n}{n^{2p}} \qquad (10)$$

On obtient des formules (9) et (10) le

Théorème 4. *Si $p$ est un entier positif et $m$ un nombre positif quelconque, la meilleure approximation asymptotique de la fonction $(1-x)^p [\lg (1-x)]^m$ vérifie l'inégalité*

$$C_p > \frac{n^{2p}}{(\lg \sqrt{2}\, n)^{m-1}} E_n \{(1-x)^p [\lg (1-x)]^m\} > \left(\frac{\pi}{4+\pi}\right)^{2p} \frac{C_p}{2\sqrt{2}} \qquad (IV)$$

où

$$C_p = m 2^{m-p+1} \int_0^\infty \frac{u^{2p-1}\, du}{e^u + e^{-u}}$$

Corollaire. Dans le cas où $p = m = 1$, l'inégalité (IV) prend la forme

$$C_1 > n^2 E_n \{(1-x) \lg (1-x)\} > \left(\frac{\pi}{4+\pi}\right)^2 \frac{C_1}{2\sqrt{2}} \qquad (IV')$$

où

$$C_1 = 2 \int_0^\infty \frac{u\, du}{e^u + e^{-u}}$$

L'inégalité (IV') démontre le théorème de M. Bernstein [3] suivant lequel l'ordre de décroissance de $E_n \{(1-x) \lg (1-x)\}$ pour $n \to \infty$ est égal à $1/n^2$.

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*MATHÉMATIQUES*

# SUR LES CHAÎNES DE BRUNS*

Par T. A. SARYMSAKOV, membre de l'Académie d'Uzbekistan

1. Considérons une suite non bornée d'épreuves indépendantes par rapport à $s$ événements $A_1, A_2, \ldots, A_s$ incompatibles et dont l'un doit nécessairement avoir lieu; les probabilités de ces événements dans chaque épreuve sont constantes et respectivement égales à $p_1, p_2, \ldots, p_s$. Nous allons supposer que $p_i > 0$ pour tous les $i$ $(i = \overline{1, s})$ ce qui ne restreint pas, évidemment, la généralité des raisonnements.

En réunissant les résultats des $r$ premières épreuves, des épreuves à partir de $k+1$ à $k+r$, de $2k+1$ à $2k+r$ *etc.*, nous obtenons une série nouvelle d'épreuves à $r$ termes, dont les résultats pour $1 \leqslant k < r$ ne sont plus indépendantes; et si $k \geqslant r$, nos épreuves à $r$ termes seront indépendantes. Introduisons pour ces épreuves à $r$ termes deux nouveaux événements: l'événement $E$ qui consiste en ce que la combinaison $A_1, A_1, \ldots, A_1$ apparaît $r$ fois et l'événement $F$ qui consiste en ce que l'une quelconque des $s^r - 1$ combinaisons à $r$ termes qui restent a lieu. Il est évident que les deux événements $E$ et $F$ sont incompatibles et l'un d'eux a nécessairement lieu, mais ils ne sont pas indépendants et nos épreuves à $r$ termes ne présentent pas une chaîne de Markoff par rapport à ces épreuves. Ce sont ces épreuves à $r$ termes par rapport aux événemens $E$ et $F$ qui forment une chaîne de Bruns. Remarquons qu'on aurait pu prendre en qualité d'événement nouveau $E$ une autre combinaison quelconque de $r$ termes et même de réunir plusieurs combinaisons à $r$ termes en une seule.

2. Désignons par $m_n$ le nombre des apparitions de l'événement $E$ dans $n$ épreuves à $r$ termes. Il est évident que la quantité aléatoire $m_n$ ne dépend pas de ce que nous considérons l'événement $F$ dans les épreuves à $r$ termes comme un événement composé ou bien nous le divisons d'abord en cas particuliers incompatibles et formant avec $E$ un groupe complet d'événements. Il est plus commode pour notre but de considérer la seconde possibilité.

Désignons par $E_2, E_3, \ldots, E_t$ $(t = s^r)$ les cas particuliers de l'événement $F$ et par $E_1$ l'événement $E$. Un calcul léger montre que les événements $E_1, E_2, \ldots, E_t$, considérés par rapport aux épreuves à $r$ termes, forment une chaîne simple homogène de Markoff. Il me semble que cette circonstance n'a pas été signalée dans la littérature. En tenant compte de cette remarque on peut, presque sans calculs, obtenir, en s'appuyant sur les résultats connus de la théorie des chaînes de Markoff, non seulements tous les résultats connus sur les chaînes de Bruns, mais encore obtenir une série de résultats nouveaux. Nous nous bornons à l'indication de deux théorèmes.

* Les résultats de cette Note ont été obtenus indépendamment et presque en même temps par V. Romanovsky.

Les probabilités $p_{ij}(i, j = \overline{1, t})$ de passer de l'événement $E_i$ à l'événement $E_j$ se déterminent facilement au moyen des probabilités $p_i$ $(i = \overline{1, s})$. Par exemple, la probabilité $p_{11}$ (c'est-à-dire la probabilité pour que l'événement $E_1$ apparaisse dans la $(l+1)$-ième épreuve à $r$ termes s'il avait lieu dans la $l$-ième épreuve à $r$ termes) est égale à $p_1^k$.

Formons la matrice $\Phi$ des probabilités $p_{ij}$, c'est-à-dire

$$\Phi = \| p_{ij} \| \qquad (i, j = \overline{1, t})$$

et écrivons l'équation caractéristique qui lui correspond

$$\Phi(\lambda) \equiv | \lambda E - \Phi | = 0 \tag{1}$$

où $E$ est la matrice unitaire.

En vertu des résultats de Romanovsky (5) on a le

Lemme. L'équation (1) a $\lambda_0 = 1$ pour racine simple qui surpasse les valeurs absolues de toutes les autres racines de cette équation et $\Phi_{ii}(1) > 0$ $(i = \overline{1, t})$, où $\Phi_{ii}(1)$ est le mineur principal du déterminant $|\lambda E - \Phi|$ qui correspond à l'élément $\lambda - p_{ii}$ pour $\lambda = 1$.

Théorème 1. *On a l'égalité*

$$Em_n = \frac{n\Phi_{11}(1)}{\sum_{\alpha=1}^{l} \Phi_{\alpha\alpha}(1)} * \tag{2}$$

Démonstration. Désignons par $M_{ij}$ l'espérance mathématique du nombre des pas nécessaires pour passer la première fois de $E_i$ vers $E_j$. On sait (6) que dans les conditions du lemme la chaîne de Markoff sera régulière, c'est-à-dire que l'on a pour cette chaîne

$$\lim_{k\to\infty} p_{ij}^{(k)} = p_j = \frac{\Phi_{jj}(1)}{\sum_{\alpha=1}^{l} \Phi_{\alpha\alpha}(1)} \qquad (j = \overline{1, t})$$

et qu'elle ne dépend pas de $i$, où $p_{ij}^{(k)}$ désigne l'élément correspondant de la matrice $\Phi^k$. On a évidemment dans ce cas

$$\lim_{n\to\infty} \pi_{ij}^{(n)} = \lim_{n\to\infty} \frac{1}{n} (p_{ij} + p_{ij}^{(2)} + \ldots + p_{ij}^{(n)}) = \frac{\Phi_{jj}(1)}{\sum_{\alpha=1}^{l} \Phi_{\alpha\alpha}(1)}$$

D'autre part A. N. Kolmogoroff (7) a démontré l'égalité suivante

$$\lim_{n\to\infty} \pi_{ij}^{(n)} = \frac{1}{M_{jj}} \qquad (j = \overline{1, t})$$

Donc nous avons

$$M_{jj} = \frac{\sum_{\alpha=1}^{l} \Phi_{\alpha\alpha}(1)}{\Phi_{jj}(1)}$$

On en déduit facilement que l'espérance mathématique de la fréquence de l'apparition de l'événement $E_j$ au cours de $n$ épreuves est égale à

$$Em_n^{(j)} = \frac{n}{M_{ij}} = \frac{n\Phi_{jj}(1)}{\sum_{\alpha=1}^{l} \Phi_{\alpha\alpha}(1)} \qquad (j = \overline{1, t}) \tag{3}$$

* $Em_n$ désigne l'espérance mathématique de $m_n$.

où $m_n^{(j)}$ est la fréquence de l'apparition de l'événement $E_j$ au cours de $n$ épreuves. Pour $j=1$ nous obtenons de la formule (3) l'égalité (2) nécessaire. V. Romanovsky m'a dit qu'en suivant cette voie on peut obtenir les formules générales pour la dispersion et les moments d'ordres supérieurs de la quantité $m_n^{(j)}$.

De la formule (3) pour les cas particuliers $s=2$, $r=2$, 3 et $k=1$, 2; $s=3$, $r=2$ et $k=1$ nous obtenons les résultats connus de Bruns ([2],[3]), Markoff ([4]) et Romanovsky ([5]). On obtient aussi facilement de cette formule une égalité asymptotique contenue dans la formule (16) de l'article de Gontcharoff ([8]). De la même manière on peut obtenir l'égalité asymptotique contenue dans la formule (28) du même article.

Théorème 2. *La distribution de la quantité* $(m_n - E_{m_n})/\sigma_{m_n}$, *où* $\sigma_{m_n}$ *est la dispersion de la quantité* $m_n$, *tend pour* $n \to \infty$ *vers la loi normale avec la moyenne nulle et la dispersion égale à* 1.

Ce théorème est un cas particulier d'un théorème plus général de Romanovsky ([6]). D'autre part le résultat du § 17 de l'article ([3]) est contenu dans le théorème 2.

Université d'État
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*FLUID DYNAMICS*

# A CASE OF UNHOMOGENEOUS TURBULENCE IN A COMPRESSIBLE FLUID

By I. A. KIEBEL, Corresponding Member of the Academy

Keller and Friedman in their outstanding contributions [1,2] have been the first to propose a method for constructing a closed system of turbulence equations. By using it Karman and Howarth [3] were able to obtain an exact solution for a very particular case, *viz.*, that of the so-called homogeneous and isotropic turbulence. The same result was obtained independently by Millionshchikov [4] and Loytzansky [5].

But no one appears so far to have given an example of solutions for the case of unhomogeneous and anisotropic turbulence. To show how a particular solution of this type of equations of Friedman and Keller can be obtained is the purpose of the present paper.

Let us consider an adiabatic motion of a heavy, ideal, compressible fluid. The equations describing this motion we shall write as follows

$$-\frac{du}{dt}=\vartheta\frac{\partial\pi}{\partial x},\quad -\frac{dv}{dt}=\vartheta\frac{\partial\pi}{\partial y},\quad -\frac{dw}{dt}=\vartheta\frac{\partial\pi}{\partial z}+g, \tag{1}$$

$$-\frac{d\pi}{dt}=(\varkappa-1)\pi\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right);\quad \frac{d\vartheta}{dt}=0 \tag{2}$$

Here $u, v, w$ are velocity components along the $x, y$ and $z$ axis, respectively; $t$ is time; $\varkappa=c_p/c_v$, specific heat ratio; $g$, acceleration of gravity; $d/dt$ indicates an individual derivative with respect to time; $\vartheta$ and $\pi$ are defined by the relations

$$\vartheta=\frac{1}{\rho}\,p^{1/\varkappa},\quad \pi=\frac{\varkappa}{\varkappa-1}\,p^{\frac{\varkappa-1}{\varkappa}} \tag{3}$$

where $\rho$, density, $p$, pressure.

We will look for such turbulent motions for which the averages are independent of time and of the horizontal coordinates $x, y$, so that

$$\varphi(x, y, z, t)=\overline{\varphi}(z)+\varphi'(x, y, z, t)$$

where $\varphi$ represents any of the five functions $u, v, w, \vartheta, \pi$; the line above a letter denotes averaging (mathematical expectation, for instance); the accent indicates the fluctuation, *i. e.* the difference between the respective function and its mean value. Put moreover $\overline{u}=\overline{v}=0$ and denote

$$\overline{w}=W(z),\qquad \overline{\vartheta}=\vartheta(z),\qquad \pi=P(z) \tag{4}$$

We introduce, further, 25 connexion moments according to the formula

$$B_{\varphi}^{\psi} = \overline{\varphi'(x-\xi, y-\eta, z-\zeta, t-\tau)\,\psi'(x+\xi, y+\eta, z+\zeta, t+\tau)} \tag{5}$$

where $\xi$, $\eta$, $\zeta$, $\tau$ are arbitrary parameters, and $\varphi'$, $\psi'$, fluctuations of any of the five functions $u$, $v$, $w$, $\vartheta$, $\pi$.

In accordance with the above assume that the $B_{\varphi}^{\psi}$ are independent of $x, y, t$ and vary only with $z$, $\xi$, $\eta$, $\zeta$, $\tau$.

According to Friedman and Keller

$$\varphi\frac{\overline{\partial\psi}}{\partial s} = \overline{\varphi}\frac{\overline{\partial\psi}}{\partial s} + \frac{1}{2}\left(\frac{\partial B_{\varphi}^{\psi}}{\partial s} + \frac{\partial B_{\varphi}^{\psi}}{\partial\sigma}\right)_0 \tag{6}$$

were $s$ stands for one of the variables $x, y, z, t$, and $\sigma$ for one of the variables $\xi, \eta, \zeta, \tau$; the subscript «0» denotes that the quantity must be taken for $\xi=\eta=\zeta=\tau=0$.

Applying the operation of averaging to both sides of (1) and (2) and making use of (6) we obtain five equations

$$\left.\begin{aligned}
&\left(\frac{\partial B_u^u}{\partial\xi}\right)_0 + \left(\frac{\partial B_v^u}{\partial\eta}\right)_0 + \left(\frac{\partial B_w^u}{\partial z} + \frac{\partial B_w^u}{\partial\zeta}\right)_0 + \left(\frac{\partial b_\vartheta^\pi}{\partial\xi}\right)_0 = 0\\
&\left(\frac{\partial B_u^v}{\partial\xi}\right)_0 + \left(\frac{\partial B_v^v}{\partial\eta}\right)_0 + \left(\frac{\partial B_w^v}{\partial z} + \frac{\partial B_w^v}{\partial\zeta}\right)_0 + \left(\frac{\partial b_\vartheta^\pi}{\partial\eta}\right)_0 = 0\\
-g = \Theta\frac{dp}{dz} + W\frac{dW}{dz} &+ \frac{1}{2}\left(\frac{\partial B_u^w}{\partial\xi}\right)_0 + \frac{1}{2}\left(\frac{\partial B_v^w}{\partial\eta}\right)_0 + \frac{1}{2}\left(\frac{\partial B_w^w}{\partial z} + \frac{\partial B_w^w}{\partial\zeta}\right)_0 +\\
&+ \frac{1}{2}\left(\frac{\partial B_\vartheta^\pi}{\partial z} + \frac{\partial b_\vartheta^\pi}{\partial\zeta}\right)_0\\
W\frac{dP}{dz} + (\varkappa-1)P\frac{dW}{dz} &+ \frac{1}{2}\left(\frac{\partial b_u^\pi}{\partial\xi}\right)_0 + \frac{1}{2}\left(\frac{\partial B_v^\pi}{\partial\eta}\right)_0 + \frac{1}{2}\left(\frac{\partial B_w^\pi}{\partial z} + \frac{\partial B_w^\pi}{\partial\zeta}\right)_0 +\\
&+ \frac{\varkappa-1}{2}\left(\frac{\partial B_\pi^u}{\partial\xi} + \frac{\partial B_\pi^v}{\partial\eta} + \frac{\partial B_\pi^w}{\partial z} + \frac{\partial B_\pi^w}{\partial\zeta}\right)_0 = 0\\
W\frac{d\vartheta}{dz} &+ \frac{1}{2}\left(\frac{\partial B_u^\vartheta}{\partial\xi} + \frac{\partial B_v^\vartheta}{\partial\eta} + \frac{\partial B_w^\vartheta}{\partial z} + \frac{\partial B_w^\vartheta}{\partial\zeta}\right)_0 = 0
\end{aligned}\right\} \tag{7}$$

We then avail ourselves of the Friedman-Keller relations

$$\left.\begin{aligned}
B_{\varphi}^{f\partial\psi/\partial s} &= \left(\frac{\partial\bar{\varphi}}{\partial s}\right)_+ B_{\varphi}^{f} + \frac{1}{2}(\bar{f})_+\left(\frac{\partial B_{\varphi}^{\psi}}{\partial s} + \frac{\partial B_{\varphi}^{\psi}}{\partial\sigma}\right),\\
B_{f\partial\psi/\partial s}^{\varphi} &= \left(\frac{\partial\bar{\psi}}{\partial s}\right)_- B_{f}^{\varphi} + \frac{1}{2}(\bar{f})\left(\frac{\partial B_{\varphi}^{\psi}}{\partial s} - \frac{\partial B_{\varphi}^{\psi}}{\partial\sigma}\right)
\end{aligned}\right\} \tag{8}$$

(like Friedman and Keller we neglect the third moments). The subscripts $+$ and $-$ mean that $x, y, z, t$ must be substituted with $x+\xi$, $y+\eta$, $z+\zeta$, $t+\tau$ or $x-\xi$, $y-\eta$, $z-\zeta$, $t-\tau$, respectively. Seeing that by the definition of the connexion moments we have $\partial B_{\varphi}^{\psi}/\partial t = B_{\partial\varphi/\partial t}^{\psi} + B_{\varphi}^{\partial\psi/\partial t}$ and that all expressions of the type $\partial\varphi/\partial t$ can be found from (1) and (2) as sums of the expressions of the type $f\cdot\partial\psi/\partial s$, therefore $\partial B_{\varphi}^{\psi}/\partial t$ as well as $\partial B_{\varphi}^{\psi}/\partial\tau$ can be expressed by means of formula (8) through the derivatives of $B_{\varphi}^{\psi}$ with respect to the coordinates and through $W$, $\Theta$, $P$.

It is natural to try to obtain the $B_{\varphi}^{\psi}$ as even functions of $\tau$. We are interested only in such values of the functions for which $\tau=0$, on account of evenness, $(\partial B_{\varphi}^{\psi}/\partial\tau)_{\tau=0}=0$*. So we can assume in our equations that $\partial B_{\varphi}^{\psi}/\partial t = \partial B_{\varphi}^{\psi}/\partial\tau = 0$.

* The results may easily be extended to the case where for $\tau=0$ the functions have their derivatives with respect to $\tau$ suffering an abrupt change.

We shall retain the notation $B_\varphi^\psi$ for $(B_\varphi^\psi)_{\tau=0}$. For convenience we introduce $z'$ and $z''$ from the relations $z'=z-\zeta$, $z''=z+\zeta$. Now from the first two equations of motion we obtain 10 equations of the form

$$2W(z')\frac{\partial B_u^\varphi}{\partial z'}-\Theta(z')\frac{\partial B_\pi^\varphi}{\partial\xi}=0,\quad 2W(z')\frac{\partial B_v^\varphi}{\partial z'}-\Theta(z')\frac{\partial B_\pi^\varphi}{\partial\eta}=0 \qquad (9)$$

where $\varphi$ can take in turn the values $u, v, \ldots, \pi$.

The third equation of motion gives 5 relations

$$\frac{\partial}{\partial z'}(W(z')B_w^\varphi)+\Theta(z')\frac{\partial B_\pi^\varphi}{\partial z'}+\dot{P}(z')B_\theta^\varphi=0 \qquad (10)$$

Here and in the sequel a point at the top of a letter denotes differentiation with respect to the argument.

The continuity equation yields five relations

$$W(z')\frac{\partial B_\pi^\varphi}{\partial z'}+(\varkappa-1)\dot{W}(z')B_\pi^\varphi+\dot{P}(z')B_w^\varphi+$$

$$+(\varkappa-1)P(z')\frac{\partial B_w^\varphi}{\partial z'}-\frac{\varkappa-1}{2}P(z')\left(\frac{\partial B_u^\varphi}{\partial\xi}+\frac{\partial B_v^\varphi}{\partial\eta}\right)=0 \qquad (11)$$

Finally we have five equations from the condition of adiabaticity

$$W(z')\frac{\partial B_\theta^\varphi}{\partial z'}+\dot{\Theta}(z')B_w^\varphi=0 \qquad (12)$$

Furthermore we must write 25 equations analogous to (9)—(12) with the substitutions $z''$ for $z'$, $\xi$ for $-\xi$, $\eta$ for $-\eta$ and $B_\varphi^\psi$ for $B_\psi^\varphi$.

Thus we have to satisfy 55 equations by means of three functions $W$, $\Theta$, $P$, and 25 connexion moments. By definition (5) the functional relations

$$B_\varphi^\psi(x, y, z, t, \xi, \eta, \zeta, \tau)=B_\psi^\varphi(x, y, z, t, -\xi, -\eta, -\zeta, -\tau) \qquad (13)$$

are to be satisfied as well.

So far we have made use only of the fact that the motion was supposed to be homogeneous in the mean over the horizontal. Now suppose it to be also isotropic in the mean with reference to horizontal directions. Denote the projection of the point $M'(x-\xi, y-\eta, z-\zeta)$ on the horizontal plane by $m'$, and the projection of the point $M''(x+\xi, y+\eta, z+\zeta)$ by $m''$. The projections of the horizontal components of motion onto the direction $\overline{m'm''}$ and the normal to it will be denoted by the letters $l$ and $n$, respectively.

Then the conditions of isotropicity will give

$$\left.\begin{aligned} B_u^u&=(B_l^l-B_n^n)\frac{\xi^2}{r^2}+B_n^n;\quad B_u^v=(B_l^l-B_n^n)\frac{\xi\eta}{r^2}\\ B_u^w&=B_l^w\frac{\xi}{r},\quad B_u^\vartheta=B_l^\vartheta\frac{\xi}{r},\quad B_u^\pi=B_l^\pi\frac{\xi}{r}\end{aligned}\right\} \qquad (14)$$

where $r^2=\xi^2+\eta^2$ and the functions $B_l^l, B_n^n, B_l^w, \ldots$ depend only on $r, z', z''$. The number of the functions to be found has been reduced to 20. All our equations are linear with respect to $B_\varphi^\psi$. Let us seek a solution for 17 $B_\varphi^\psi$ in the form of a product of functions of $r$ with functions of $z'$ and $z''$

$$B_\varphi^\psi=b_\varphi^\psi(z', z'')f_\varphi^\psi(r) \qquad (15)$$

Substituting (15) in the 50 equations (9)—(12) and making use of (14) we shall find after some simple, though rather cumbersome, computations that all $f_\varphi^\psi$ are expressed explicitly by means of Bessel functions $J_0(\beta r)$, $J_1(\beta/r)$, where $\beta$ is an arbitrary parameter whose dimension is length$^{-1}$. As for the $b_\varphi^\psi$ these 17 functions can all be expressed through four, *viz.* $b_\vartheta^\vartheta$, $b_n^n$, $b_l^\vartheta$, $b_\vartheta^l$. The latter functions must satisfy a system of 8 partial differential equations.

Carrying further the analysis and using (13) we shall find it possible to represent the last four functions of $z'$ and $z''$ as a combination of two one-argument functions $\tilde{u}$, $\tilde{\vartheta}$ satisfying two differential equations

$$\frac{d}{dz}\left(\frac{W^2\tilde{\vartheta}}{\Theta}\right)-\tilde{\vartheta}P-\frac{2\sqrt{2}}{3}\Theta\frac{d}{dz}\left(\frac{W\tilde{u}}{\Theta}\right)=0$$

$$W\frac{d}{dz}\left(\frac{W\tilde{u}}{\Theta}\right)+(\varkappa-1)\frac{WW'}{\Theta}\tilde{u}+\frac{\varkappa-1}{4}3^2\tilde{u}P-\frac{3}{2\sqrt{2}}\left[\frac{WP}{\Theta}\tilde{\vartheta}+\right.$$

$$\left.+(\varkappa-1)P\frac{d}{dz}\left(\frac{W\tilde{\vartheta}}{\Theta}\right)\right]=0 \qquad (17)$$

To these equations we have to add those obtained from (7) after the $B_\xi^\eta$ are substituted therein. Then two of the equations (7) will be satisfied, and the remaining three give

$$g=\Theta\dot{P}-W\dot{W}-\frac{3}{2\sqrt{2}}\frac{W}{\Theta}\dot{\tilde{\vartheta}}\tilde{u}+\frac{1}{2}\frac{d}{dz}\left(\frac{W}{\Theta}\dot{\tilde{\vartheta}}\right)^2+\frac{2\sqrt{2}}{3}\tilde{\vartheta}\frac{d}{dz}\left(\frac{W}{\Theta}\dot{\tilde{u}}\right) \qquad (18)$$

$$W\dot{P}+(\varkappa-1)P\dot{W}-2(\varkappa-2)\frac{W}{\Theta}\frac{d}{dz}\tilde{u}^2-\frac{2\sqrt{2}}{3}\frac{W\dot{\tilde{\vartheta}}}{\Theta}\frac{d}{dz}\left(\frac{W}{\Theta}\tilde{u}\right)-$$

$$-\frac{2\sqrt{2}}{3}(\varkappa-1)\frac{W\dot{\tilde{u}}}{\Theta}\frac{d}{dz}\frac{W\dot{\tilde{\vartheta}}}{\Theta}=0 \qquad (19)$$

$$W\dot{\Theta}-\frac{3}{\sqrt{2}}\tilde{u}\tilde{\vartheta}-\frac{W}{\Theta}\dot{\tilde{\vartheta}}^2=0 \qquad (20)$$

After the five functions $W$, $\Theta$, $P$, $\tilde{u}$, $\tilde{\vartheta}$ are established from the five equations (16)—(20), all the $B_\xi^\eta$ can be obtained from the following relations

$$B_l^l=\frac{2}{\beta}\tilde{u}(z')\tilde{u}(z'')\frac{dJ_1(\beta r)}{dr}\ ;\ B_n^n=\frac{2}{\beta}\tilde{u}(z')\tilde{u}(z'')\frac{1}{r}J_1(\beta r)$$

$$B_l^w=\frac{\sqrt{2}}{3}\tilde{u}(z')\frac{W(z'')}{\Theta(z'')}\tilde{\vartheta}(z'')J_1(\beta r);\ B_l^\vartheta=-\frac{\sqrt{2}}{3}\tilde{u}(z')\tilde{\vartheta}(z'')J_1(\beta r)$$

$$B_l^u=-\frac{4}{\beta}\tilde{u}(z')\frac{W(z'')}{\Theta(z'')}\tilde{u}(z'')J_1(\beta r)$$

$$B_w^w=\frac{W(z')W(z'')}{\Theta(z')\Theta(z'')}\dot{\tilde{\vartheta}}(z')\dot{\tilde{\vartheta}}(z'')J_0(\beta r);\ B_w^\vartheta=-\frac{W(z')}{\Theta(z')}\dot{\tilde{\vartheta}}(z')\tilde{\vartheta}(z'')J_0(\beta r)$$

$$B_w^u=-\frac{2\sqrt{2}}{\beta}\frac{W(z')W(z'')}{\Theta(z')\Theta(z'')}\dot{\tilde{\vartheta}}(z')\dot{\tilde{u}}(z'')J_0(\beta r)$$

$$B_\vartheta^\vartheta=\tilde{\vartheta}(z')\tilde{\vartheta}(z'')J_0(\beta r);\ B_\vartheta^u=\frac{2\sqrt{2}}{3}\tilde{\vartheta}(z')\frac{W(z'')}{\Theta(z'')}\dot{\tilde{u}}(z'')J_0(\beta r)$$

$$B_u^u=-\frac{8}{3^2}\frac{W(z')}{\Theta(z')}\frac{W(z'')}{\Theta(z'')}\dot{\tilde{u}}(z')\dot{\tilde{u}}(z'')J_0(\beta r)$$

The remaining $B_\xi^\eta$ will be found from these relations by virtue of (13).

The established equations can be employed in solving the problem of turbulence arising in a still air over a non-uniformly heated horizontal surface and also (for $g=0$) in the problem of diffusion of a shock wave.

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*PHYSIQUE MATHÉMATIQUE*

# SOLUTION DU PROBLÈME DE REFROIDISSEMENT D'UN SEMI-ESPACE OCCUPÉ PAR UNE MATIÈRE A $n$ POINTS CRITIQUES DONT LES INTERVALLES SONT CARACTÉRISÉS PAR DES PROPRIÉTÉS PHYSIQUES DIFFÉRENTES

Par G. P. IVANTZOV

*(Présenté par S. L. Sobolejf, de l'Académie, le 1. IX. 1944)*

La solution connue de la congélation d'une couche du sol d'une épaisseur illimitée (un point critique) due à Stephan et exposée parmi autres par Frank[1], Gröber et Erk [2], a été appliquée par Sohwarz [3,4] pour le calcul de la cristallisation du métal dans les lingotières. Ici nous allons montrer que cette solution peut être généralisée pour le refroidissement d'un semi-espace occupé par une matière à $n$ points critiques, correspondant aux $n$ températures critiques, le passage desquelles est suivi de l'absorption ou du dégagement de chaleur. Supposons que dans les intervalles entre les températures critiques les propriétés thermophysiques de la matière restent constantes, mais qu'elles ne sont pas les mêmes dans des intervalles différents.

La condition initiale est

$$\tau = 0 \quad t = t_{n+1}$$

Pour conditions limites de semi-espace l'on a

$$\left.\begin{matrix} x = 0 \\ \tau > 0 \end{matrix}\right\} \; t = t_0 \qquad t_0 < t_{n+1}$$
$$x = \infty \qquad t = t_{n+1}$$

Les conditions limites aux températures critiques sont

$$t_{\xi_1 - 0} = t_1; \; t_{\xi_1 + 0} = t_1, \; \ldots, \; t_{\xi_n - 0} = t_n, \; t_{\xi n + 0} = t_n$$
$$q_{\xi_1 - 0} = q_{\xi_1 + 0} + Q_1 \frac{d\xi_1}{d\tau}, \; \ldots, \; q_{\xi_n - 0} = q_{\xi_n + 0} + Q_n \frac{d\xi_n}{d\tau}$$

Ici $q$, courants thermiques, $Q$, chaleurs de transformation. Nous avons donc $2 + 2n + n = 3n + 2$ conditions limites.

Formons un système de $n+1$ équations différentielles de Fourier suivant le nombre des intervalles entre les températures limites et critiques

$$\frac{\partial t_i}{\partial \tau} = a_i \frac{\partial^2 t_i}{\partial x^2}, \; i = 1, 2, \ldots, n, \; n+1$$

Le déplacement des surfaces aux températures critiques sera défini par les équations

$$\xi_k = m_k \sqrt{\tau}, \quad k = 1, 2, \ldots n$$

Le champ de température pour chaque intervalle sera

$$t_i = C_i + D_i G\left[\frac{x}{\sqrt{4a_i\tau}}\right], \qquad i = 1, 2, \ldots, n, n+1$$

où $G$ désigne une fonction de Kramp.

Il est à déterminer $2n+2$ coefficients $C$ et $D$ et $n$ coefficients $m$, en tout $3n+2$ coefficients.

Puisque l'on a $3n+2$ conditions limites, le système a une solution déterminée.

En posant $Q_K = 0$ on peut résoudre le problème du refroidissement d'un massif dont les propriétés thermophysiques sont liées à la température par une dépendance graduelle.

En posant $n = 2$ et $Q_1 = Q_2 = 0$ nous pouvons résoudre le problème de la cristallisation d'un alliage binaire, formant des solutions solides; dans ce cas, le processus de cristallisation peut être approximativement considéré comme refroidissement d'une matière ayant une chaleur spécifique élevée; la conductibilité dans l'intervalle de cristallisation peut être prise en moyenne entre les conductibilités calorifiques des phases solide et liquide.

En posant encore dans le problème précédent $Q_1 \neq 0$, nous avons la solution du problème de la cristallisation d'un alliage binaire dont les composants forment des mélanges à l'état solide (et ne forment pas de solutions solides).

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*PHYSICS*

# STEPPED IONIZATION OF HYDROGEN BY ELECTRONIC IMPACT

By B. YAVORSKY

*(Communicated by S. I. Vavilov, Member of the Academy, 3. VIII. 1944)*

The following expression gives the differential cross-section corresponding to the ejection from an atom of an electron of a definite energy $E_k = k^2h^2/8\pi^2m$ within a solid angle $d\sigma$ in the direction $(\chi, \psi)$ with respect to the initial direction of the incident electron

$$I_k d\sigma\, d\omega\, dk = \frac{4\pi^2m^2}{h^4}\frac{k'}{k}\left|\int\int V\psi_{n_1lm}\psi_\varkappa\, e^{i(\vec{kn_0}-\vec{k'n_1})\vec{r}}dv\,dv_1\right|^2 \times d\sigma\, d\omega\, dk \qquad (1)$$

This incident electron is scattered within a solid angle $d\omega$ and its impulse changes from $hk/2\pi$ to $h(k+dk)/2\pi$.

Applying Bethe's formula ([2]) to the integration of $v_1$ we can obtain

$$I_k d\sigma\, d\omega\, dk = \frac{64\pi^2m^2}{h^4}\frac{k'}{k}\frac{\varepsilon^4}{K^4}\left|\int \psi_{n_1lm}\psi_\varkappa e^{i(\vec{kn_0}-\vec{k'n_1})\vec{r}}dv\right|^2 d\sigma\, d\omega\, dk. \qquad (2)$$

Here $\varepsilon$ is the charge of the electron; $m$, its mass, $K^2 = k^2 + k'^2 - 2kk'\cos\delta$; $\delta$, the scattering angle of the incident electron; $\psi_{n_1lm}$, the wave function of the initial state of the atom, which for an arbitrary $S$-state has the form

$$\psi_{n_100} = N_{n_100}L'_{n_1}\left(\frac{2rZ}{n_1a_0}\right)e^{-\frac{Zr}{n_1a_0}} \qquad (3)$$

where

$$N^2_{n_100} = \frac{1}{n_1\pi}\frac{(n_1-1)!}{(n_1!)^3}\left(\frac{Z}{n_1a_0}\right)^3 \qquad (3')$$

$Z$ is the charge of the nucleus; $a_0$, the Bohr first radius; $\psi_\varkappa$, the wave function of a $\varkappa$-state of the continuous spectrum of hydrogen, has, according to Sommerfeld ([3]), the form

$$\psi_\varkappa(r) = \varkappa\left[\frac{n'}{\pi\left(1-e^{-2\pi n'}\right)}\right]^{1/2}\frac{e^{i\varkappa r}}{\Gamma(1+n)}\int_0^\infty u^n e^{-u}I_0\{2(i\varkappa u)^{1/2}\xi'^{1/2}\}\,du, \qquad (4)$$

where

$$\xi' = r(1+\cos\Theta); \quad n = Z/ia_0\varkappa; \quad n' = in$$

The value of $\cos\Theta$ can be expressed in the following form [1]

$$\cos\Theta = \cos\vartheta\cos\chi + \sin\vartheta\sin\chi\cos(\Phi - \psi)$$

we have here $(k\vec{n}_0 - k'\vec{n}_1)\vec{r} = K_1\cos\vartheta$. It is necessary to compute the integral

$$W = \int\int \exp\left\{\frac{Zr}{n_1a_0} + i(k\vec{n}_0 - k'\vec{n}_1)\vec{r} + ixr - u\right\} L'_{n_1}\left(\frac{2rZ}{n_1a_0}\right) u^n I_0\{2(ix\xi' u)^{1/2}\}\, dv\, du \quad (5)$$

Making use of the expansion of Lagger's polynomials [4]

$$L'_{n_1}(x) = (-1)^{n_1}\left\{x^{n_1} - \frac{n_1(n_1+1)}{1}x^{n_1-1} + \frac{n_1(n_1-1)(n_1+1)n_1}{1\cdot 2}x^{n_1-2} + \ldots \right.$$
$$\left. \ldots + (-1)^{n_1}(n_1+1)!\right\} \quad (6)$$

we get

$$W = (-1)^{n_1}\sum_{\tau=0}^{\tau=n_1} A_\tau \int\int \exp\left\{-\frac{Zr}{n_1a_0} + i(k\vec{n}_0 - k'\vec{n}_1)\vec{r} + ixr - u\right\} \times$$
$$\times r^{n_1-\tau}u^n I_0\{2(ix\xi' u)^{1/2}\}\, dv\, du \quad (7)$$

where

$$A_\tau = \left(\frac{2Z}{n_1a_0}\right)^{n_1-\tau}\frac{n!}{\tau!(n_1-\tau)!}\frac{(n_1+1)!}{(n_1+1-\tau)!}$$

Introducing in computation the parabolic coordinates $\xi = r(1+\cos\vartheta)$; $\eta = r(1-\cos\vartheta)$; $\Phi$ and denoting $Z/n_1a_0 = \mu$ we shall have an integral of the form

$$W = (-1)^{n_1}\sum_{\tau=0}^{\tau=n_1}\frac{A_\tau}{2^{n_1-\tau+2}}\int\int \exp\left\{-\frac{\mu}{2}(\xi+\eta) + \frac{1}{2}iK(\xi-\eta) + \frac{ix}{2}(\xi+\eta) - u\right\} \times$$
$$\times u^n(\xi+\eta)^{n_1-\tau+1} I_0\{2(ix\xi' u)^{1/2}\}\, du\, dv \quad (8)$$

where $\xi'$, $\eta'$ are parabolic coordinates of the direction of ionization.

If $\vartheta$ be the angle between the direction in which the electron is ejected and the vector $k\vec{n}_0 - k'\vec{n}_1$, we can use the relation (1)

$$\xi' = \xi\cos^2\frac{\vartheta}{2} + \eta\sin^2\frac{\vartheta}{2} + 2(\xi\eta)^{1/2}\cos\frac{\vartheta}{2}\sin\frac{\vartheta}{2}\cos\Phi$$

The theory of Bessel functions contains the proof of the following relation (see [5], p. 361)

$$I_0\{2(ix\xi' u)^{1/2}\} = I_0\left\{2\left(uix\eta\sin^2\frac{\vartheta}{2}\right)^{1/2}\right\} I_0\left\{2\left(uix\xi\cos^2\frac{\vartheta}{2}\right)^{1/2}\right\} \quad (9)$$

Its substitution into (8) and integration over $\Phi$ gives

$$W = (-1)^{n_1}\sum_{\tau=0}^{\tau=n_1}\frac{A_\tau\pi}{2^{n_1+1-\tau}}\int\int\int \exp\left\{\frac{\xi}{2}(-\mu + iK + ix) + \frac{\eta}{2}(-\mu - iK + ix) - u\right\} \times$$
$$\times u^n(\xi+\eta)^{n_1+1-\tau} I_0\left\{2\left(ix\eta u\sin^2\frac{\vartheta}{2}\right)^{1/2}\right\} \times$$
$$\times I_0\left\{2\left(ix\xi u\cos^2\frac{\vartheta}{2}\right)^{1/2}\right\} d\xi\, d\eta\, du \quad (10)$$

Developing $(\xi+\eta)^{n_1+1-\tau}$ as a Newton's binomial let us make use of Hankel's formula (see [5], p. 393)

$$\int_0^\infty e^{-p^2t^2} I_\nu(at)\, t^{\Omega-1}dt = \frac{\Gamma\left(\frac{\nu+\Omega}{2}\right)\left(\frac{1}{2}\frac{a}{p}\right)^\nu}{2p^\Omega\Gamma(\nu+1)} \cdot e^{\frac{a^2}{4p^2}} F\left(\frac{1}{2}\nu-\frac{1}{2}\Omega+1;\quad \nu+1;\quad \frac{a^2}{4p^2}\right) \tag{11}$$

Then integration with respect to $\xi$ and $\eta$ gives

$$\frac{\Gamma(n_1-\tau-\beta+2)}{(\mu-iK-ix)^{n_1-\tau+2-\beta}} \exp\left(-\frac{2\cos^2\frac{\vartheta}{2}\, ixu}{\mu-iK-ix}\right)\times$$

$$\times F\left(-(n_1-\tau-\beta+2)+1;\ 1;\ \frac{2\cos^2\frac{\vartheta}{2}\, ixu}{\mu-iK-ix}\right) \tag{12}$$

$$\frac{\Gamma(\beta+1)}{(\mu+iK-ix)^{\beta+1}} \exp\left(-\frac{2\sin^2\frac{\vartheta}{2}\, ixu}{\mu+iK-ix}\right)\times$$

$$\times F\left(-(\beta+1)+1;\ 1;\ \frac{2\sin^2\frac{\vartheta}{2}\, ixu}{\mu+iK-ix}\right). \tag{13}$$

Thus we have

$$W=(-1)^{n_1}\sum_{\tau=0}^{n_1}\sum_{\beta=0}^{n_1+1-\tau} \frac{\tau A_\tau B_\beta \Gamma(\beta+1)\,\Gamma(n_1-\tau-\beta+2)}{2^{n_1+1-\tau}(\mu+iK-ix)^{\beta+1}(\mu-iK-ix)^{n_1-\tau-\beta+2}}\times$$

$$\times\int_0^\infty \exp\left\{-u\left(\frac{2\sin^2\frac{\vartheta}{2}\, ix}{\mu+iK-ix}+\frac{2\cos^2\frac{\vartheta}{2}\, ix}{\mu-iK-ix}+1\right)\right\} u^n F F' du. \tag{14}$$

Here $B_\beta$ are the binomial coefficients.

By disclosing the hypergeometrical functions the integral with respect to $u$ can be reduced to the following form

$$(\mu+iK-ix)^3(\mu-iK-ix)^3\int_0^\infty \exp\left\{-u\left[\frac{\mu^2+x^2-2xK\cos\vartheta+K^2}{(\mu-ix)^2+K^2}\right]\right\}\times$$

$$\times\frac{u^n du}{[\mu^2+x^2+K^2-(2xK\cos\vartheta-2x^2-2ix\mu)\,u+(x^2+x^2\cos^2\vartheta)\,u^2]^3} \tag{15}$$

Finally, introducing a new variable

$$\left(\frac{x^2+x^2\cos^2\vartheta}{\mu^2+x^2+K^2}\right)^{1/2} u = t \tag{16}$$

and denoting

$$2\lambda = \frac{(2xK\cos\vartheta-2x^2-2ix\mu)\,(\mu^2+x^2+K^2)^{1/2}}{(\mu^2+x^2+K^2)\,(x^2+x^2\cos^2\vartheta)^{1/2}}$$

$$b = \frac{(\mu^2+x^2-2xK\cos\vartheta+K^2)\,(\mu^2+x^2+K^2)^{1/2}}{[(\mu-ix)^2+K^2]\,(x^2+x^2\cos^2\vartheta)^{1/2}} \tag{17}$$

we obtain an integral of the following form

$$\int_0^\infty e^{-bt}(1-2\lambda t+t^2)^{-3}t^n dt$$

By the definition of Hugenbauer's polynomials

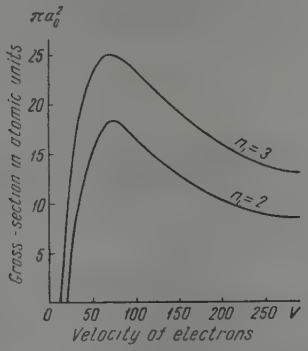
$$(1-2\lambda t+t^2)^{-\nu}=\sum_{S=0}^{\infty} C_S^{\nu}(\lambda)\, t^S \tag{18}$$

Then using a well-known formula of the integral calculus

$$\int_0^{\infty} e^{-\alpha t} t^k\, dt=\frac{(k+1)!}{\alpha^{k+1}}$$

we have the result of integration with respect to $u$

$$\frac{[(\mu-ix)^2+K^2]^3}{(\mu^2-x^2+K^2)^3}\left(\frac{\mu^2+x^2-K^2)}{x^2-x^2\cos^2\vartheta)}\right)^{\frac{n_1+1}{2}}\sum_{S=0}^{\infty} C_S^3(\lambda)\,\frac{n_1+S+1)!}{b^{n_1+S+1}} \tag{19}$$




Computed cross-sections for ionization of hydrogen. Stepped ionization for $n_1=2$ and $n_1=3$ levels

Ultimately we obtain

$$W=(-1)^{n_1}\sum_{\tau=0}^{n_1}\sum_{\beta=0}^{n_1+1+\tau}\sum_{S=0}^{\infty}\frac{\pi A_{\tau}B_{\beta}\Gamma(\beta+1)\,\Gamma(n_1-\tau-\beta+2)\,[(\mu-ix)^2+K^2]^3}{2^{n_1+1-\tau}(\mu-ix+iK)^{\beta+1}(\mu-ix-iK)^{n_1-\tau-\beta+2}}\times$$

$$\times\frac{(\mu^2+x^2+K^2)^{\frac{n_1+1}{2}}\,C_S^3(\lambda)\,(n_1+S+1)!}{(\mu^2+x^2+K^2)^3\,(x^2+x^2\cos^2\vartheta)^{\frac{n_1+1}{2}}\,b^{n_1+S+1}} \tag{20}$$

The further computation of the effective cross-section is carried out by substituting the obtained result in (1) and by numerical integration over the angles of scattering and ionization. The figure presents the results for the cases $n_1=2$ and $n_1=3$.

The stepped ionization of hydrogen has never been studied directly by experiment. By comparing the computations performed with the calculation of the straight ionization from the ground state [1] we are enabled to compare the probabilities for straight and stepped processes of ionization.

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*GEOPHYSICS*

# ON THE RESULTS OF RADIO-OBSERVATIONS DURING THE SOLAR ECLIPSE (CORPUSCULAR AND ULTRA-VIOLET) OF JULY 9, 1945

By J. L. ALPERT and B. N. GOROZHANKIN

*(Communicated by N. D. Papalexi, Member of the Academy, 28. VIII. 1945)*

The present paper is a brief account of the results of radio-observations carried out by the authors during the total solar eclipse of July 9, 1945, while working with an expedition fitted out by the L. I. Mandelstam Laboratory of Oscillations, Institute of Physics of the Academy of Sciences of the USSR, under the guidance of N. D. Papalexi.

Systematic investigations of the ionosphere during solar eclipses carried out during the past 10—15 years in various countries have made it possible to establish that the ultra-violet radiation of the Sun, being the chief ionizing agent of the $E$-layer, has also an essential effect on the $F_1$-layer. As for the rôle of the corpuscular radiation of the Sun in the ionization of the upper layers of the atmosphere, many workers were concerned with this important problem during previous eclipses but no definite results were obtained.

As Chapman[1] and Appleton[2] pointed out, there is a time lag between the corpuscular and photon eclipses. Neither do the phases of the corpuscular eclipse coincide for particles with different velocities. Owing to the latter circumstance the influence of the ultra-violet eclipse as well as corpuscular can be traced during solar eclipses for particles moving with different speed. As to the corpuscular eclipse, the attention of previous investigators was focussed mainly on particles moving at a rate of about 1500—1600 kilometres per second, which is the most probable velocity according to Milne[3].

The conditions on July 9, 1945 were very favourable for radio-observations in this country, for not only was the total phase of ultra-violet eclipse in the ionosphere observable in the daytime at a number of points, including the neighbourhood of Moscow, but there was also crossing between the zones of corpuscular eclipse for neutral particles of different velocities.

The main purpose of our experiments was to elucidate the rôle of the ultra-violet and corpuscular radiation of the Sun in ionizing the upper layers of the atmosphere, as well as to investigate the possible bending of the reflecting regions of the ionosphere during photon eclipse[4].

Apparatus. The observations were carried out by the impulse method. The impulse transmitter, a 0.5-kW unit, was set up in the Institute of Physics of the USSR Academy of Sciences in Moscow. An Γ-shaped antenna served as the aerial; the steadiness of the radiation was controlled with an oscillograph. The duration of the impulses (which had a frequency of 50 cycles per sec.) was about $10^{-4}$ sec.

The receiving appliance which was set up south-west of Moscow, 37.4 km away, consisted of an Adcock direction finder of the semi-stationary type with a

superheterodyne receiver. An RCA-905 oscillograph served as the indicator; to one pair of its plates the rectified intermediate frequency voltage from the receiver was applied and to the other, the tooth-shaped voltage of the time base, synchronized with an alternating current of 50 cycles per sec. To the modulator of the oscillograph were led the sharp voltage peaks («time marks»), likewise synchronized with the alternating current network, whose frequency was controlled with a frequencymeter. Observations consisted in measuring the time lag of the impulses and in determining the direction (azimuth) of their arrival in a horizontal plane. Precision in determining the time lag, expressed in length, was $\pm(6 \div 7)$ km, while precision in reading the azimuth on the goniometer of the direction finder was $\pm(2 \div 5)°$.

Systematic observations were carried out daily on two fixed frequencies, on 3.8 and 6.0 megacycles per sec. from July 4 through July 14, and on 6 and 6.85 megacycles per sec. from July 18 through July 27, from 8 a. m. to 8 p. m. at intervals of half an hour for a period of 10 min. on each frequency. On July 9 and 10 continuous observations were carried out from 5 to 6 p. m. on the one frequency of 3.8 megacycles per sec.

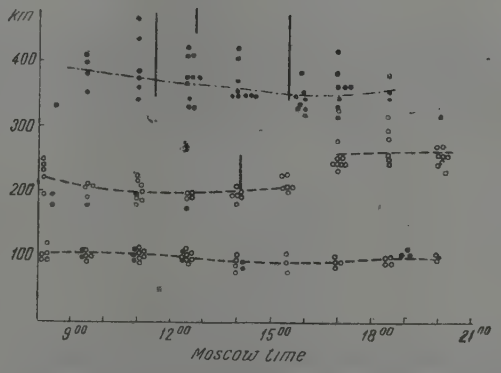


Fig. 1. Results of measurements of effective paths of reflections from the ionosphere on the frequency 3.8 megacycles per sec. between July 4 and July 14, 1945, inclusive (circles and dotted lines); and on the frequency 6 megacycles per sec. between July 4 and July 27, 1945, inclusive (points and dotted lines). The vertical straight lines denote continuous group reflections

Experimental Results. On the frequency 3.8 megacycles (Fig. 1) regular reflections were observed: up to 3 in the afternoon from the $E$ and $F_1$-layers and beginning with 5 in the afternoon from the $F_2$-layer. On the frequency 6 megacycles per sec. (Fig. 1) the reflections from the $E$ and $F_1$-layers were only sporadic-while from the $F_2$-layer reflections of an extraordinary ray were frequently obser, ved whose critical frequency was close to 6 megacycles per sec. Owing to the fact that during the second half of the experiments (from July 18 through July 27) an increase in the critical frequency for the $F_2$-layer was observed, observations were also carried out in this period on the frequency 6.85 megacycles, whose conditions for reflection were nearly the same as the conditions for reflection for the frequency 6 megacycles per sec. in the first half of the experiments (up to July 14). Here individual group reflections were observed now and then, a large part of which coincided with magnetic disturbances of short duration. Throughout the time we were running off the experiments there were magnetically active days in the period from July 4 through July 7 and from July 16 through July 18; on the day of the eclipse, July 9, the state of the magnetic field of the earth remained completely undisturbed. Neither were there any perturbations in solar activity observed rthoughout the period we were conducting the experiments.

Results of observations made on July 9 on the frequency 3.8 megacycles per sec. are given in Fig. 2, where the azimuth of the impulses reflected from the $F_2$-layer have been plotted in the upper part. The figure reveals complete cessation of reflections from the $E$-layer during the period of ultra-violet eclipse and their reappearance only at 46 min. past 5 in the afternoon, *i. e.* 21 min. after the maximum phase of photon eclipse*. Characteristic of the reflections from the $F_2$-layer at that time was their great steadiness and appreciably more than usual intensity. Furthermore, double reflections were also observed. Between 2 and 4 p. m. a thunder storm over the observation point considerably interfered with the measurements.

Between 3.30 and 3.40 p. m. reflections with a group lag corresponding to an altitude of 350 km, *i. e.* the $F_2$-layer, appeared. This was the only time in

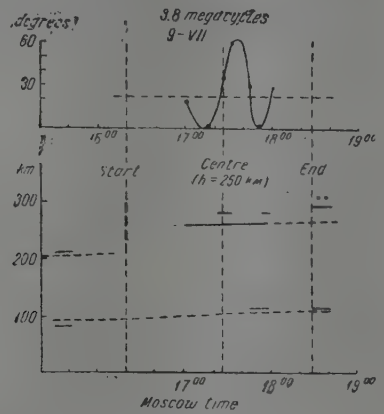


Fig. 2. Results of observations on July 9 on the frequency 3.8 megacycles per sec. The dotted lines stand for mean values for control days, solid lines represent the results of measurements made on July 9. Azimuths of the arrival of waves reflected from the $F_2$-layer are shown above. Each point is the result of the averaging of the bearings for a time interval of 2 ÷ 3 min.

all our experiments that we observed reflections from the $F_2$-layer on the frequency 3.8 megacycles per sec. at this time of the day. From Fig. 2 it is also evident that during ultra-violet eclipse there was a change in the azimuth of the signals reflected from the $F_2$ -layer, which afterward reverted to its original value not coinciding with the direction toward the transmitter. The azimuth of the reflections from the $E$-layer had likewise a value different from the direction towards the transmitter, which was also observed on control days.

The results of observations made on July 9 on the frequency 6 megacycles per sec. are given in Fig. 3. At 10 o'clock very intensive reflections of the usual type from the $F_2$-layer, the $E$-layer and an intermediate region between the $E$ and $F_1$-layers (at an altitude approaching 150 km), as well as group reflections, appeared. At 11.30 a. m. and then again at 1.00 and 4.00 p. m. reflections of the usual type and group reflections were observed throughout each series of observations. Similar reflections occurred at 8.30 p. m. as well. No noticeable change in the azimuth was observed on this frequency.

Here it would be well to note the following circumstances: a) up to July 10 in-

* The maximum phase for the $E$-layer over the observation point was at 5.25 p. m.

clusive reflections from the $F_2$-layer were never observed until 11.00 a. m. Only in the second half of our experiments, owing to the enhanced ionization at the time, were they observed at earlier hours as well; b) in the entire month group reflections from $E$—$F_1$ were observed only on July 9; c) group reflections from the $F_2$-layer were observed very rarely (5—6 times in all); on July 9 they were detected all day long; d) the thunder-storm that swept over the observation point on July 9

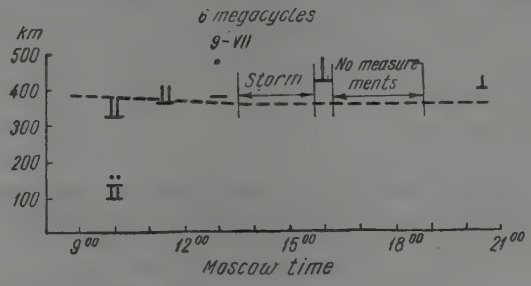


Fig. 3. Results of observations on July 9 on the frequency 6 megacycles per sec. The dotted line stands for the mean values for control days. The vertical solid lines denote continuous group reflections. The horizontal solid lines denote that there were reflections throughout the entire series of observations. The points stand for unsteady reflections

and the strong atmospherics that made their appearance even before it, made it impossible to carry out all the observations in the interval between 2.00 and 4.00 p.m.

## Preliminary Analysis of Results

Ultra-violet Eclipse. The measurements on the frequency 3.8 megacycles per sec. (Fig. 1) show that during ultra-violet eclipse the degree of ionization of the $E$-region decreases. At 5.46 p. m. the critical frequency for the $E$-region passed right through the value 3.8 megacycles per sec. Then the degree of ionization apparently assumed the usual value for that time.

The values we obtained for the azimuths of the signals reflected from the $E$-region make it possible to assume deviation of their paths, evoked by bending of the reflecting layer*. Indirect confirmation of this may be obtained from an approximate evaluation of the factor $I_0/\alpha k^2$ introduced by Appleton(2) to characterize the different regions of the ionosphere (where $I_0$ characterizes the intensity of the solar radiation and is the number of electrons forming in 1 cm³ at midday at the equator at the equinox, $\alpha$ is the coefficient of recombination and $k = 1.24 \cdot 10^{-8}$). This factor is obtained from the value of the critical frequency 3.8 megacycles per sec. for 5.46 p. m., taking into account the latitude, the extent to which the solar disc is observed and the non-uniformity of the ultra-violet radiation from the uneclipsed part of the solar disc. We found it to be more than three times as large as the value expected on the basis of data corresponding to the given phase of solar activity(2). This may be explained by the fact that the zenith angle $\varphi$ of an arriving signal at the point of reception was considerably greater than in the case of regular reflection, and the effective value of the critical frequency was equal not to 3.8 but to 3.8 cos $\varphi$ megacycles per sec.

Space does not permit us to dwell in greater detail on this subject and also on

* The correctness of this assumption will be checked later against the results of the experiments now in progress on the direction of the arrival of impulses reflected from various regions of the ionosphere.

the observed bending of the reflecting $F_2$-region during ultra-violet eclipse which, as our measurements show, occurred in the lower part of this layer.

Corpuscular Eclipse. The very intensive protracted group reflections from the $F_2$-layer observed only on July 9 (in the absence of magnetic and other disturbances) on the frequency 6 megacycles per sec. were in the nature of a perturbation piercing the entire ionosphere from the $F_2$-layer down to the $E$-layer. They should apparently be tied in with the corpuscular eclipse that occurs at the same time for neutral particle velocities of the order of 500 km/sec. The results of our observations would seem to indicate, in our opinion, that the corpuscular radiation of the Sun is not the agent determining the fundamental state of ionization of any of the regions of the ionosphere, and that its action is purely perturbative in nature.

Conclusions. The investigations we have carried out make it possible to draw the following conclusions:

1. It is confirmed that the ultra-violet radiation of the Sun determines the fundamental state of ionization of the $E$-layer. The measured values of the azimuths of the reflections from the $F_2$-layer make it possible to draw the conclusion that the bending of the latter occurs during ultra-violet eclipse.

2. From the results of our experiments it may be concluded that the corpuscular radiation of the Sun, in particular for particles with velocities of the order of 500 km/sec., affects the ionosphere.

In conclusion we wish to express our sincere gratitude to N. D. Papalexi, Member of the Academy, for his undivided attention to our work and a number of valuable suggestions, and also to G. I. Hofman and A. A. Ainberg for their essential help in carrying out the experiments.

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*TECHNICAL PHYSICS*

# NYQUIST DIAGRAMS FOR A THOMPSON SYSTEM WITH TWO DEGREES OF FREEDOM AND THEIR PHYSICAL INTERPRETATION

By K. TEODORCHIK

*(Communicated by B. A. Vvedensky, Member of the Academy, 8. XII. 1944)*

I. Cross Coupling System (Fig. 1, *a*). If this kind of circuit is switched over according to Nyquist [1] for taking (computing) his diagram, we shall obtain the circuit represented in Fig. 1, *b*. At the input a sinusoidal voltage of variable frequency $p$ is applied

$$u = u_0 \sin pt \tag{1}$$

Denoting the output voltage by $x$, the voltage at the terminals $C_1$ by $y$, and assigning the valve a cubic characteristic we shall have

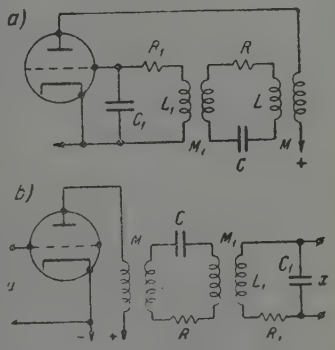


Fig. 1

$$x = \frac{Q_1}{C_1}; \quad y = \frac{Q}{C}$$

$$L\ddot{Q} + R\dot{Q} + \frac{1}{C}Q + M_1\ddot{Q}_1 = -M(s - s_2u^2)\dot{u}$$

$$I_a = su - \frac{1}{3}s_3u^3$$

$$M\ddot{Q} + L_1\ddot{Q}_1 + R_1\dot{Q}_1 + \frac{1}{C_1}Q_1 = 0 \tag{2}$$

or, if partial frequencies and coefficients of damping and coupling are introduced,

$$\nu_1^2 = \frac{1}{L_1C_1}; \quad \nu^2 = \frac{1}{LC}; \quad \delta_1 = \frac{R_1}{2L_1}; \quad \delta = \frac{R}{2L}; \quad \alpha_1 = \frac{M_1C}{L_1C_1}; \quad \alpha = \frac{M_1C_1}{LC}$$

$$\alpha_1\ddot{x} + \ddot{y} + 2\delta_1\dot{y} + \nu_1^2 y = -M\nu_1^2(s - s_2u^2)\dot{u}$$

$$\ddot{x} + 2\delta\dot{x} + \nu^2 x + \alpha\ddot{y} = 0 \tag{3}$$

Thus we have a problem of two coupled oscillatory circuits under the action of an external periodical force

$$E = -M\nu_1^2\left(s - \frac{1}{4}s_2u_0^2\right)pu_0\cos pt + \frac{1}{4}M\nu_1^2ps_2u_0^3\cos 3pt. \tag{4}$$

Neglecting the third harmonic and putting

$$E = iu_0Pe^{ipt}; \quad x = Xe^{ipt}; \quad y = Ye^{ipt}$$

we obtain for the complex coefficient of amplification

$$\mu^* = \frac{X}{u_0} = \frac{\alpha p^2 P}{\{(\nu^2 - p^2) + 2ip\delta\}\ \{(\nu_1^2 - p^2) + 2ip\delta_1\} - \alpha\alpha_1 p^2} = \mu e^{i\psi}$$

Denoting the modulus of the denominator by $w$ we find

$$\sin\psi = \frac{(\nu^2 - p^2)(\nu_1^2 - p^2) - \alpha\alpha_1 p^4 - 4\delta\delta_1 p^2}{w}$$

$$\cos\psi = \frac{2p[\delta(\nu_1^2 - p^2) - \delta_1(\nu^2 - p^2)]}{w} \qquad (5)$$

$$\mu = \frac{-\delta M p^3 \nu_1^2 \left(s - \frac{1}{4} s_2 u_0^2\right)}{w}$$

Note that the expression in the numerator of $\sin\psi$ has its roots exactly equal to two frequencies of the auto-oscillatory system in Fig. 1 (2). Therefore the intersection points of the Nyquist diagram with the real axis correspond to

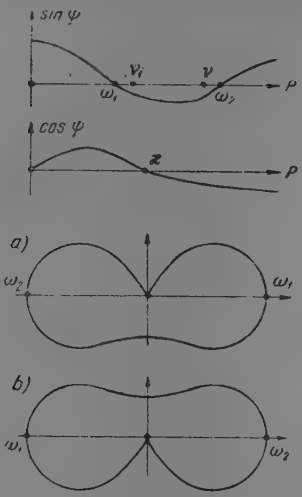


Fig. 2

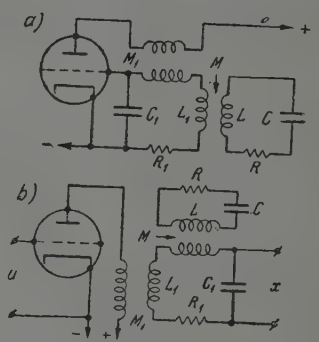


Fig. 3

the two auto-oscillation frequencies permitted by the system. Denoting these frequencies by $w_1$ and $w_2$ respectively, we can write

$$\sin\psi = \frac{(1 - \alpha\alpha_1)(p^2 - \omega_1^2)(p^2 - \omega_2^2)}{w} \qquad (6)$$

$$\cos\psi = \frac{2p(p^2 - x^2)}{w}$$

where

$$x = \pm\sqrt{\frac{\delta\nu_1^2 + \delta_1\nu^2}{\delta + \delta_1}}$$

It will be obvious that as $p$ grows from 0 to $\infty$ the signs before $\sin\psi$ and $\cos\psi$ change as shown in Fig. 2.

The Nyquist diagram of the system under investigation is easily built further on the basis of (5), (6) and Fig. 2. For $M < 0$ we obtain the Fig. 2, *a* and for $M > 0$ the Fig. 2, *b*.

From the Nyquist diagrams we have built it may be inferred, in accordance with the analytical theory of the circuit represented in Fig. 1, that generation is possible with a positive as well as negative feed-back. The sign of

the feed-back determines whether the frequency $\omega_1$ or $\omega_2$ will be alone generated.

II. Let us now consider the scheme of a generator loaded with a linear circuit (Fig. 3, *a*). Adapting it to the computation of Nyquist's diagram we obtain the circuit represented in Fig. 3, *b*.

Using the same notation as in the case I we get

$$\ddot{x} + 2\delta_1\dot{x} + \nu_1^2 x + \alpha_1 \ddot{y} = -M_1\nu_1^2 p u_0 \left(s - \frac{1}{4} s_2 u_0^2\right) \cos pt$$

$$\alpha\ddot{x} + \ddot{y} + 2\delta\dot{y} + \nu^2 y = 0 \qquad (7)$$

Introducing again

$$E = -M_1\nu_1^2 p u_0 \left(s - \frac{1}{4} s_2 u_0^2\right) i e_{ipt} = i u_0 P e^{ipt} \qquad (8)$$

we have

$$\mu^* = \frac{P\{(\nu^2 - p^2) + 2ip\delta\}\{(\nu^2 - p^2)(\nu_1^2 - p^2) - \alpha\alpha_1 p^4 - 4p^2\delta\delta_1 - 2ip[\delta_1(\nu^2 - p^2) + \delta(\nu_1^2 - p^2)]\}}{[(\nu^2 - p^2)(\nu_1^2 - p^2) - \alpha\alpha_1 p^4 - 4p^2\delta\delta_1]^2 + 4p^2[\delta_1(\nu^2 - p^2) + \delta(\nu_1^2 - p^2)]^2}$$

Denoting the denominator in this expression by the letter $w_1$ we get

$$\mu \sin\psi = \frac{P(\nu^2 - p^2)\left[(\nu_1^2 - p^2)(\nu_1^2 - p^2) - \alpha\alpha_2 p^4 + 4p^2\delta^2 \frac{\nu_1^2 - p^2}{\nu^2 - p^2}\right]}{w_1} \qquad (9)$$

$$\mu \cos\varphi = \frac{2P\delta_i\left\{(\nu^2 - p^2)^2 + 4p^2\delta^2 + \alpha\alpha_1 p^4 \frac{\delta}{\delta_1}\right\}}{w_1} \qquad (10)$$

In the numerator of the expression for $\mu \sin\psi$ the factor in the brackets is again the equation of frequencies of the investigated oscillatory system ([2,3]).

According to the magnitude of the coupling this equation has

1) one real positive root if $\alpha\alpha_1 < 4\delta^2/\nu^2$,

2) three roots if $\alpha\alpha_1 > 4\delta^3/\nu^2$.

If in either case we investigate the variation of $\sin\psi$ and $\cos\psi$ as functions of the frequency $p$ and find the real value of the amplification coefficient under the frequencies permitted by the system, we shall be able to plot a Nyquist diagram.

For $M < 0$ we obtain Fig. 4, *a* in the case of one root, and Fig. 4, *b* or *c* in the case of three roots. For $M < 0$ the feed-back is positive for all frequencies, and each of them can be self-excited if the given oscillation is stable both in amplitude and frequency (phase).

The stability conditions for the case $\psi = \psi(p)$ are obtained in the form

$$\left(\frac{\partial\mu}{\partial u_0}\right)_a < 0, \quad \left(\frac{d\psi}{dp}\right)_\omega < 0 \qquad (11)$$

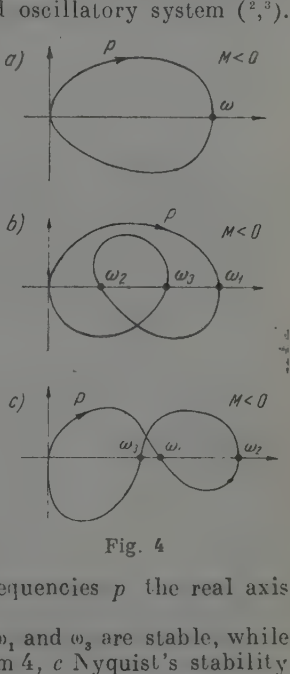

Fig. 4

The first condition is always fulfilled for soft systems (see below), the second requires that on moving in the Nyquist diagram toward increasing frequencies $p$ the real axis $\mu$ be intersected from above downwards.

From Fig. 4, *b* it may be seen that the frequencies $\omega_1$ and $\omega_3$ are stable, while the frequency $\omega_2$ is unstable. Applying to the diagram 4, *c* Nyquist's stability criterion we shall find that in the latter case only the frequency $\omega_3$ can be generated.

In a non-linear system both the amplification coefficient $\mu$ and the phase angle $\psi$ are variable nor merely with the frequency $p$, but also with the out-

put amplitude $u_0$. Therefore a non-linear system is characterized by a family of Nyquist diagrams depending on a single parameter $u_0$ rather than by one diagram. If the coefficient of amplification decreases monotonously with increasing $u_0$, this family in general is uniquely and entirely comprised within the $\mu^*$ $(0, p)$ Nyquist diagram. Such system may be described as soft. If, on the other hand, the coefficient of amplification first increases with increasing $u_0$ and then drops down to zero, the system is said to be rigid.

The condition for the self-excitation of the system can be written as follows

$$\psi(0, p) = 0, \quad \mu(0, p) > 1 \tag{12}$$

In a soft system in the case of self-excitation the Nyquist diagram representing the system begins to contract toward the origin of coordinates and this continues until the steady state characterized by the conditions of stationary amplitude and frequency is attained

$$\psi(a, \omega) = 0, \quad \mu(a, \omega) = 1 \tag{13}$$

For a rigid system the family of Nyquist diagrams in a portion of the plane becomes a two-valued function of the amplitude. In this case if the point $\mu = +1$ lies in the two-valued region of the diagram, a rigid excitation of auto-oscillations takes place.

On the basis of these non-linear considerations can be explained the stabilization of a softly self-exciting system observed experimentally when an external harmonic force of a sufficiently great amplitude is applied at the output (asynchronous damping of auto-oscillations). And they also account for the self-excitation of a stable rigid system under an external force whose amplitude lies within certain limits (threshold and ceiling of asynchronous self-excitation) ([4],[5]).

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*CHEMISTRY*

# DEARYLATION OF NON-SYMMETRICAL ORGANIC TIN COMPOUNDS AS A METHOD OF SYNTHESIZING $RSnX_3$ IN THE ALIPHATIC SERIES

By M. E. PAVLOVSKAYA and K. A. KOZESHKOV

*(Communicated by A. N. Nesmeyanov, Member of the Academy, 23. III. 1945)*

There are simple methods of synthesizing compounds of the type $RSnX_3$ in the aromatic series (1). In the aliphatic series, however, no direct and general methods have been devised, with the result that even the first members of the series have been incompletely described and there is disagreement among authors regarding their physical constants, or else the latter drop out completely from the general system of properties of the given class of compounds.

Thus, for example, the melting point of methyl-tin trichloride, $CH_3SnCl_3$, the simplest representative of $RSnX_3$, is reported as 105—107° by Pope and Peachy(2), who obtained it from methylstannic acid and hydrogen chloride (resp., phosphorus trichloride); 40° by Druce(3), who obtained it from methylstannic acid and hydrochloric acid; 43° by Pfeiffer and Lehnardt(4), who obtained it frommethylstannic acid dried at 100° and dry hydrogen chloride, and 46° by Pfeiffer and Halperin (5) later.

Likewise in striking discord with the relatively low melting points observed usually in this class of compounds is the value 310° that Druce(6)reports for ethyl-tin tribromide, which is at sharp variance with the values reported for its immediate homologues (53° for methyl-tin tribromide and 112° for isopropyl-tin tribromide).

It was necessary, therefore, to find a direct, general method, different from Meyer's(7)(through the alkylstannic acids), of synthesizing $RSnX_3$ (especially the chlorides and bromides) in the aliphatic series (about the synthesis of corresponding iodides see (2,8)).

Developing the method of obtaining aryl-tin trihalides proposed by one of us(9) and based on the action of excess tetrahalogenides of tin on tetra-aryl tin and taking into account the selective detachment of aryl groups in non-symmetrical organic tin compounds, we arrived at a reaction that takes place according to the following equations

$$Ar_3SnX + RMgX \rightarrow Ar_3SnR + MgX_2 \qquad (I)$$

$$Ar_3SnR + 3\ SnX_4 \rightarrow RSnX_3 + 3\ ArSnX_3 \qquad (II)$$

(where R is an aliphatic radical). Applied to the methyl compound it leads to methyl-tin-trichloride with a melting point 45-46° that agrees with the value given by Pfeiffer and Halperin(5). Upon arylation it gives methyltriphenyl tin (identical to the compound obtained by a different method).

Since with the aid of a Grignard reagent any aliphatic radical retaining upon splitting off a linkage with the $SnX_3$ residue, may be introduced into $Ar_3SnX$ (equation I), the reaction (II) may serve as a direct general method of synthesizing organic tin chlorides (resp. bromides) of the type $RSnX_3$ in the aliphatic series.

## Experimental Part

Methyltriphenyl tin. To a Grignard reagent prepared from 42.6 g (0.3 mol) of methyd iodide and 7.2 g (0.3 at) of metallic magnesium in 150 $cm^3$ of ether and filtered through a thoroughly dry funnel provided with a porous glass plate (No. 2), 73.3 g (0.2 mol) of dry finely powdered triphenyl-tin fluoride were added in small portions([10]). The reaction that takes place is betrayed by the dissolution of the monofluoride, which is sparingly soluble in ether. When all the triphenyl-tin fluoride had been added, the solution was boiled about an hour. Then the reaction product was decomposed by a solution of ammonium chloride (at 0°) and the ether layer separated and dried over calcium chloride. Upon filtering and distilling off the ether, the oil that remained gradually crystallized out completely. After recrystallization from thoroughly dried petroleum ether, 54.2 g of methyltriphenyl tin were obtained with a melting point of 60°, the value reported by Krause and Schmitz([11]). The yield was 74 per cent of the theoretical (computed in terms of the monofluoride).

Methyl-tin trichloride. 36.5 g (0.1 mol) of methyltriphenyl tin (see above) was mixed while cooling with 78.3 g (0.3 mol) of anhydrous tin chloride. The mixture was then heated in a sealed tube at 185—190° for a period of 1—1.5 hrs. (It is essential that heating be uniform, otherwise partial resinification appears at the sites of overheating). The slightly brown liquid that was obtained, consisting of a mixture of methyl-tin trichloride and phenyl-tin trichloride, was subjected to distillation. Upon driving off the tin chloride that did not enter into the reaction, we collected a fraction that cools down into a crystal mass in the receiver. Afrer recrystallization from thoroughly dried petroleum ether, the substance was found to melt at 45—46° (the melting point of methyl-tin trichloride reported by Pfeiffer and Halperin is 46° ([5])). The yield was 15.6g (65 per cent of the theoretical, equation (II). The residue in the flask upon distillation in vacuum, boiled at 142—143° (25 mm Hg). This is phenyl-tin trichloride.

Phenylation of methyl-tin trichloride. To an ether solution of phenyl-lithium obtained as usual from 4.71 g (0.3 mol) of bromobenzene and 0.23 g (0.03 at. plus 10 per cent) of metallic lithium and filtered off from excess of the metal (in an atmosphere of nitrogen), an ether solution of 2.16 g (0.01 mol minus 10 per cent) of methyl-tin trichloride obtained as above, was gradually added. After heating for a period of half an hour, treating with a solution of ammonium chloride (at 0°), separating and drying the ether layer, distilling off the ether and recrystallizing the residue from thoroughly dried petroleum ether, 2.7 g of methyltriphenyl tin (82 per cent of the theory) were obtained with a melting point of 60°. A mixed melting point test made with the methyltriphenyl tin described at the beginning of the experimental part showed no depression in the value.

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*CHIMIE PHYSIQUE*

# SUR LE SPECTRE D'ABSORPTION DU BENZÈNE ADSORBÉ PAR LE GEL DE SILICE

Par E. N. PAVLOVA

*(Présenté par A. N. Terenin, de l'Académie, le 12. I. 1945)*

Les spectres d'absorption des substances adsorbées, analysés par de-Boer([1]), ainsi que par d'autres auteurs([2],[3]), consistent d'une région large d'absorption continue dont le maximum est dévié par rapport à celui d'absorption de la même substance dans sa phase gazeuse. La valeur et le signe de la déviation sont fonctions de la situation réciproque des surfaces d'énergie potentielle des molécules excitée et normale. On a été conduit à admettre que le meilleur moyen d'interpréter le spectre de la substance adsorbée est celui de le comparer au spectre de la même substance dans des solvants différents.

Dans un certain nombre des cas on a déterminé à l'aide de cette méthode comparative le caractère de la liaison de la molécule avec la surface. Mais grace à l'absorption continue les changements du spectre d'absorption, notés par les auteurs ci-dessus mentionnés, ne nous permettent pas d'admettre une solution unique du problème le plus important, tel que celui des changements des liaisons dans la molécule elle-même sous l'influence du champ de surface. De plus, à mesure que les bandes d'absorption deviennent plus larges et diffuses dans tous les cas analysés une disparition totale de la structure d'oscillation du spectre de la substance adsorbée a lieu et l'absorption augmente dans la région des ondes longues en effectuant un déplacement du maximum d'absorption vers le côté du rouge, ce qui peut masquer totalement le déplacement spontané des niveaux électroniques de la molécule dans le champ de surface ([3],[4]). Pour cette raison il ne serait pas dénué d'intérêt d'analyser un système analogue adsorbant—adsorbé où la réaction réciproque avec la surface n'entraînerait pas la disparition de la structure d'oscillation dans le spectre d'absorption, et de déterminer ainsi les changements spécifiques des fréquences de la molécule soumise à l'action de l'adsorption. Ce sont en premier lieu les molécules peu actives qui conservent la structure d'oscillation pendant l'adsorption moléculaire. C'est pour cette raison que nous avons choisi pour notre expérience le benzène adsorbé par le gel de silice.

Sans compter que la transparence soit absolument nécessaire dans la région à analyser du spectre, le gel de silice fut admis comme adsorbant en raison de la tendance d'employer la méthode spectrale pour l'analyse des phénomènes qui ont lieu sur la surface des adsorbants employés en pratique.

Le gel de silice préparé par le mode de Fells et Firth([5]) était soumis à une dialyse prolongée et à la dessiccation. La plaque du gel de silice séché était placée dans l'appareil et était chauffée préalablement au vacuum à la température de 500—700° C pendant quelques heures. Le spectre du benzène adsorbé était analysé à la température ordinaire dans la région des pressions de vapeur du benzène à partir de $p=0.038$ mm Hg jusqu'à $p=2.9$ mm Hg. Les essais effectués parallèlement (mesurant les isothermes à l'aide de la balance de Mac-Bain) ont démontré que ces pressions correspondent au remplissage de la surface de $q=0.007$ g jusqu'à $q=0.044$ g pour 1 g de gel de silice. Dans ces conditions le spectre du benzène adsorbé est composé de quatre bandes qui correspondent aux plus intenses ban-

des de vapeur B, C, D et E*, élargies et déviées vers le rouge. Aucun changement spécifique des fréquences dans le spectre du benzène adsorbé n'a été démontré.

Dans le premier tableau nous avons présenté les longueurs d'onde des bandes B, C, D, E du benzène adsorbé, vaporisé, liquide et dissous; dans le second, les valeurs du déplacement des bandes du benzène liquide, dissous et adsorbé et aussi du benzène solide, par rapport aux bandes de sa vapeur, d'après les données de Kroneberger et de Pringsheim (6).

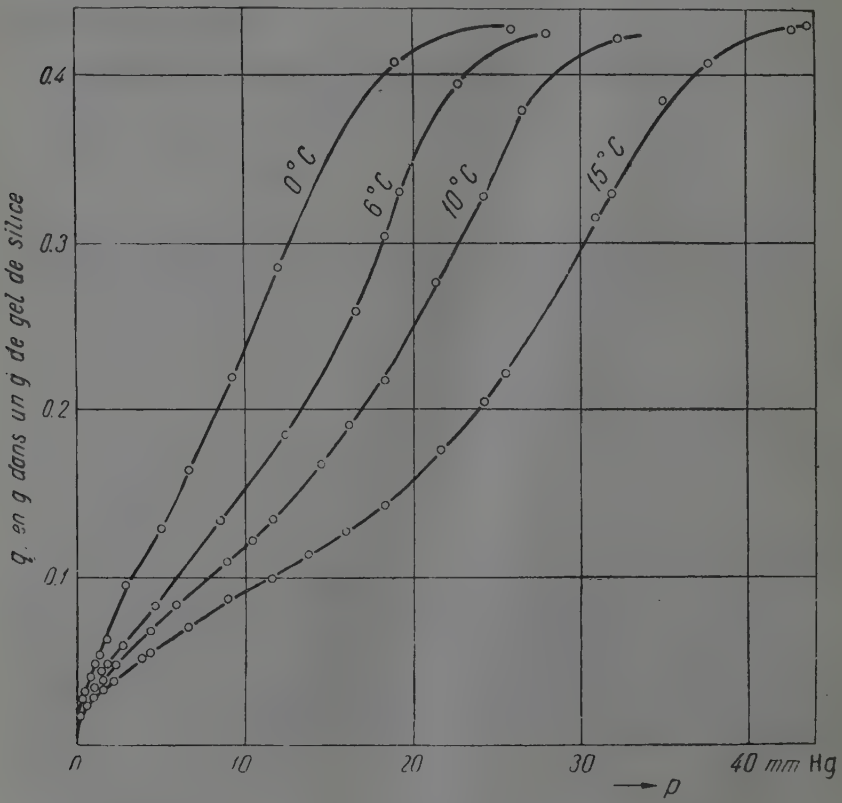


Fig. 1

Selon les tableaux 1 et 2 il est évident que le spectre du benzène adsorbé occupe une place intermédiaire entre le spectre de la vapeur et celui du liquide (une déviation de 117 $cm^{-1}$ vers le rouge par rapport au spectre de la vapeur est à noter) et qu'il correspond au spectre du benzène dissous dans l'eau dans les limites de l'erreur possible. Ce dernier fait nous conduit à admettre que le champ de surface du gel de silice séché au maximum, où se trouvent les molécules adsorbées du benzène, est analogue à celui qui est formé par les molécules d'eau de la solution. Il est à noter que dans les conditions de vacuum permanent et du lavage par la vapeur du benzène un chauffage à 700°C prolongé et fait à plusieurs reprises rend impossible la présence des molécules d'eau sur la surface du gel de silice. Il ne reste que de l'eau constitutionnelle qu'on ne peut éloigner sans nuire à la structure. Au point de vue des conceptions appliquées ordinairement à l'ad-

* Les bandes indiquées d'après Henri (9).

Tableau 1

| | B | | C | | D | | E | |
|---|---|---|---|---|---|---|---|---|
| | λ Å | ν cm$^{-1}$ | λ Å | ν cm$^{-1}$ | λ Å | ν cm$^{-1}$ | λ Å | ν cm$^{-1}$ |
| Benzène adsorbé . . . . | 2598 | 38471 | 2537 | 39398 | 2479 | 40327 | 2424 | 41240 |
| » vaporisé . . . | 2591 | 38588 | 2530 | 39514 | 2472 | 40446 | 2416 | 41371 |
| » liquide . . . . | 2613 | 38254 | 2551 | 39189 | 2491 | 40134 | 2436 | 41048 |
| » dissous dans l'eau. . . . . | 2600 | 38477 | 2538 | 39382 | 2481 | 40299 | 2425 | 41221 |

Tableau 2

Déplacement en cm$^{-1}$ par rapport aux bandes de la vapeur

| | B | C | D | E |
|---|---|---|---|---|
| Benzène adsorbé . . . . . | 117 | 116 | 119 | 129 |
| » liquide . . . . . | 334 | 325 | 312 | 323 |
| » dissous dans l'eau . . . . . | 111 | 132 | 147 | 150 |

Benzène solide d'après les données de Kroneberger et Pringsheim (⁶), 285 cm$^{-1}$.

sorption des vapeurs il est assez important que les données spectrales signalent l'absence du benzène liquide dans les pores du gel de silice à l'intervalle du remplissage ci-dessus cité; ce qui prouve une fois de plus qu'on n'a pas le droit d'adopter la théorie de Polanyi pour la région des remplissages faibles. Malgré ce fait les données de nos mesures coïncident bien avec la courbe caractéristique de Polanyi, et ceci ne fait que souligner le caractère formel de sa théorie.

Nous avons mesuré les isothermes d'adsorption de benzène par le gel de silice à températures différentes dans tous les intervalles de pression jusqu'à celle de la pression de saturation (fig. 1). Les parties initiales des isothermes sont bien formulées par l'équation de Freundlich: $q = 27.6 p^{0.41}$, où $q$, la quantité de benzène adsorbé en mg pour 1 g de gel de silice; $p$, la pression en mm Hg jusqu'au remplissage de l'ordre $q = 0.045$ g pour 1 g de gel de silice, ce qui correspond au début de la condensation capillaire. Lorsque le remplissage est au delà de 0.045 g, deux phénomènes démontrent la condensation capillaire: la coïncidence complète des courbes de structure à températures différentes et la divergence des courbes de sorption et de désorption (formant, il faut le dire, une courbe de hystérésis peu accentuée) qui est manifestée à peu près en présence des remplissages pareils et qui est caractéristique pour la condensation capillaire (⁷,⁸). A notre grand regret il ne nous a pas réussi à démontrer à l'aide de spectroscopie l'apparition du liquide dans les pores de l'adsorbant dès le début de la condensation capillaire, c'est-à-dire pendant les remplissages au delà de 0.045 g pour 1 g de gel de silice à cause d'un grand accroissement de la couche adsorbante et de la fusion complète des bandes.

Nous avons calculé à l'aide de l'équation $d \lg n/d(1/T) = -Q/R$ la chaleur d'adsorption $Q$ du benzène sur le gel de silice pendant les différents remplissages de la surface. La courbe présentée sur la figure 2 démontre un maximum accentué au moment où $q = 0.047$ g pour 1 g de gel de silice, au delà duquel la chaleur d'adsorption diminue lentement et s'approche à celle de l'évaporation du benzène.

Selon notre avis cette allure de la courbe est due à l'action réciproque des molécules adsorbées dans la première couche. Le potentiel supplémentaire de van der Waals, qui en résulte, doit augmenter à mesure du remplissage de la première couche en s'ajoutant au potentiel de surface, ce qui doit provoquer à son tour l'accroissement de la chaleur d'adsorption que nous signalons dans la région des remplissages faibles.

En ce qui concerne les couches suivantes il est évident que leur énergie d'adsorption doit être plus petite que celle de la première couche. Ce fait et aussi la

condensatiôn capillaire doivent mener à une diminution de la chaleur d'adsorption aussitôt que la première couche est remplie. En pratique il est probable qu'un ménisque se forme dans les pores fines dans le cas où la structure de l'adsorbant du type de gel de silice est poreuse, hétérogène et, par conséquent, la condensation capillaire a lieu avant que la première couche soit remplie complètement. Ceci nous explique la diminution de la chaleur d'adsorption qui commence lorsque le remplissage est égal à $\sim$0.047 g pour 1 g de gel de silice, ce qui correspond à $\sim$30 pour 100 de la première couche entière (la surface de 1 g de gel de silice calculée d'après la condensation capillaire est $4 \cdot 10^6$ cm$^2$).

Nous sommes loin de donner pour l'unique notre explication de l'allure de la couche de la chaleur d'adsorption. Cependant si elle correspond plus ou moins à

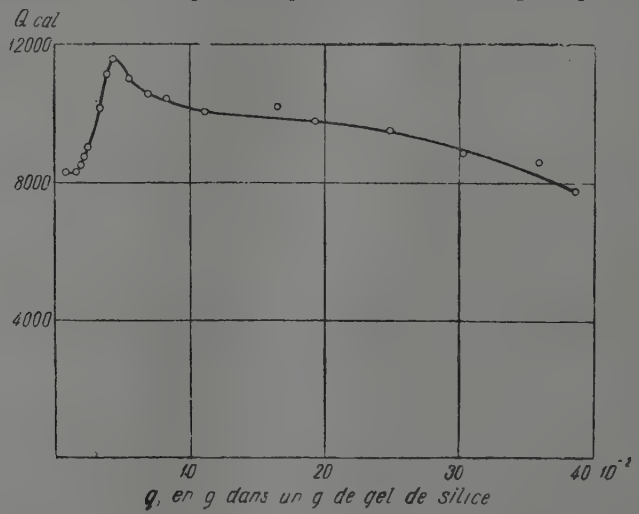


Fig. 2

la réalité, nous devons peut être tâcher d'expliquer les écarts de l'isotherme de Langmuir, transformée dans nos essais en l'isotherme de Freundlich, plus par l'action mutuelle des molécules dans la couche adsorbée, moins par ce fait que la surface elle-même de l'adsorbant n'est pas homogène au point de vue énergétique.

Il nous reste à noter que les phénomènes de la structure d'oscillation dans le spectre d'absorption du benzène adsorbé nous permettent de juger de l'état de l'adsorbé dans les pores de l'adsorbant en dispersion fine et nous démontre que le champ de surface est analogue au champ du solvant polaire en ce qui concerne le cas considéré de la sorption moléculaire. Il est à douter si le manque de changements dans les fréquences caractéristiques de la molécule pendant l'adsorption du benzène, dû à l'action mutuelle faible avec la surface, soit spécifique pour la sorption de cohésion.

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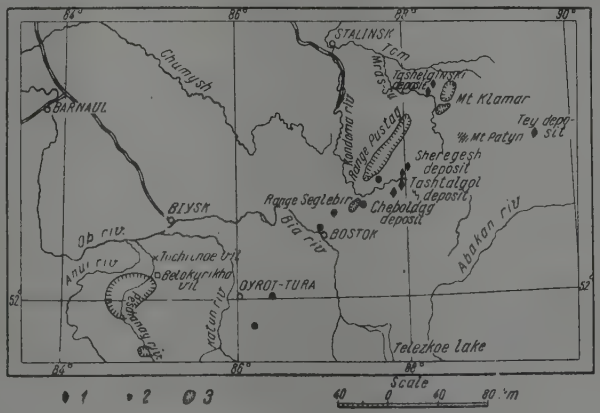
*GEOCHEMISTRY*

# A CONTRIBUTION TO THE PETROCHEMICAL CHARACTERISTIC OF THE GRANITOIDS OF THE IRON-ORE REGIONS OF THE KUZNETSK ALATAU*

By P. I. LEBEDEV, Corresponding Member of the Academy

The granite intrusions of the western slope of the Kuznetsk Alatau, in the areas of development of which the zones of iron-ore deposits of Mountain Shoria have been formed, are considered by some of the investigators of Western Siberia as belonging to different magmatic cycles: according to G. A. Kuznetzov(1), the Porozhinski granites on the Mras-su River are of Upper Silurian age, the Pustag granites




*1*, iron-ore deposits; *2*, points of manganese mineralization; *3*, general outlines of massifs

are Lower Silurian in age; according to M. A. Ussov(2), the Porozhinski granites are of Algonkian age, and the Pustag granites, as well as the intrusions of Telbes and Temirtau, are Lower Devonian.

According to M. A. Ussov (3), while the age of the granite intrusions of the two above regions (the Mras-su region, the Porozhinski granites, and in the Kondoma

* By a petrochemical characteristic is meant a characteristic of the chemical composition of the igneous rocks, consisting of data of routine quantitative analysis, supplemented by data of spectrum analysis, having the value of semiquantitative determinations. In order to make them more demonstrative the latter are denoted by 2 conventional symbols. The analysis and comparison of the petrochemical characteristics of the igneous rocks of the same or of different petrographic formations permit not only of certain conclusions which are of a petrogenetic significance (the trend of the differentiation of contamination processes), but also of a characteristic of their metallogenic peculiarities.

region, the granites of the Pustag Range) is different, the iron-ore deposits within them are genetically related to the same Lower Devonian phase of magnetite mineralization.

A detailed study of the chemical properties of the representatives of the separate granite intrusions of the so-called Porozhinski granites(4) on the Mras-su River (the Tashelga group of iron-ore deposits), on the one hand, and of the granites of the region of the Kondoma deposits, on the other, gives closely similar petrochemical characteristics, obtained by chemical and spectrum analysis (Tables 1 and 2).

Table 1

Petrochemical Characteristic of Granitoids of the Kondoma Group, Mountain Shoria (Kuznetzk Alatau)*

| | I | II | III | IV | V | VI | VII |
|---|---|---|---|---|---|---|---|
| $SiO_2$ | 74.30 | 74.30 | 62.52 | 64.28 | — | — | 74.01 |
| Zr | +++ | +++ | +++ | ++ | ++ | ++++ | — |
| $Al_2O_3$ | 13.26 | 13.67 | 15.42 | 17.76 | — | — | 14.23 |
| Be | ++ | ++ | ++ | — | + | ++ | — |
| $TiO_2$ | 0.23 | 0.19 | 0.67 | 0.43 | — | — | 0.25 |
| $Fe_2O_3$ | 1.46 | 0.40 | 2.12 | 1.75 | — | — | 0.22 |
| Cr | + | — | — | — | ++++ | — | — |
| V | + | — | +++ | — | +++ | ++ | — |
| FeO | 0.33 | 1.01 | 3.27 | 1.44 | — | — | 1.17 |
| MnO | 0.09 | 0.06 | 0.08 | 0.10 | — | — | 0.05 |
| MgO | 0.16 | 0.15 | 3.11 | 0.70 | — | — | 0.14 |
| Ni, Co | + | — | ++ | — | ++++! | — | — |
| CaO | 0.34 | 0.94 | 4.86 | 1.07 | — | — | 0.76 |
| BaO | | +++ | 0.14 | ++ | ++ | +++ | — |
| Sr | | ++ | — | ++ | +++ | +++ | — |
| $Na_2O$ | 3.76 | 3.88 | 3.72 | 5.37 | — | — | 4.24 |
| $K_2O$ | 4.92 | 4.86 | 2.48 | 5.87 | — | — | 4.52 |
| $P_2O_5$ | — | — | 0.14 | 0.21 | — | — | — |
| Sn | +++ | + | ++ | — | + | ++ | — |
| Mo | + | — | + | — | — | ++ | — |
| W | — | — | — | — | — | — | — |
| Pb | ++ | + | + | — | —! | ++ | — |
| Cu | | | | — | | | — |
| $H_2O_{110}$ | 0.28 | 0.36 | 0.20 | — | — | — | 0.20 |
| $H_2O$ | 0.73 | 0.11 | 1.34 | 0.88 | — | — | 0.29 |
| Total | 99.86 | 99.93 | 100.07 | 99.86 | — | — | 100.08 |

* I—Granite of the region of Kondomsky village (V. A. Egorov, analyst); II—granite of Mt. Sarlyk; III—adamellite of Temirtau (V. A. Egorov, analyst); IV—adamellite of Mt. Kubbes (V. A. Egorov, analyst); V—hornblendic granite, summit of Mt. Uchtash; VI—granite of the region of Kedrovka village; VII—granite of the summit of the Pustag Range (V. A. Moleva, analyst).

Note. + denotes traces of lines; ++, very weak lines; +++, weak spectral lines; ++++, spectral lines of medium intensity;—denotes the absence of the element.

In addition to the close analogy in the magmatic characteristic of the two granite groups ($\alpha$, $RO/R_2O$, $Na_2O/K_2O$), the following data may be pointed out: (1) the granites of Mt. Uchtash (Kondoma River) are characterized by the presence of Cr, V, Ni, CO. (2) In some intrusives (Kubbess, Temirtau) an increased $P_2O_5$ content is noted. (3) The granitoids of Temirtau (adamellites), directly associated with iron-ore deposit, are characterized by the presence of V, Ni, Co, while Cr

Table 2

Petrochemical Characteristic of the Granitoids of the Regions of the Mras-su River and the Tey Deposit*

| | Porozhinski granites | | | Region of the Tey iron-ore deposit | |
|---|---|---|---|---|---|
| | I | II | III | IV | V |
| $SiO_2$ | 75.30 | | | 69.93 | 65.60 |
| Zr | +++ | +++ | +++ | +++ | ++++ |
| $Al_2O_3$ | 13.06 | — | — | 15.97 | 18.19 |
| Be | ++ | + | ++ | ++ | ++ |
| $TiO_2$ | 0.16 | — | very intensely | 1.40 | 0.88 |
| $Fe_2O_3$ | 0.69 | — | — | 0.99 | 1.79 |
| FeO | 0.51 | — | — | 0.90 | 1.17 |
| Cr | +++ | — | — | +++ | — |
| V | ++ | ++ | ++ | — | + |
| MnO | 0.03 | ++++ | — | 0.10 | 0.03 |
| Ni, Co | — | — | — | — | — |
| MgO | 0.30 | — | — | 0.60 | 0.74 |
| CaO | 0.97 | — | — | 2.22 | 0.40 |
| BaO | +++ | +++ | +++ | + | +++0.04 |
| Sr | ++ | ++ | +++ | — | +++ |
| $Na_2O$ | 3.74 | — | — | 3.84 | 8.56 |
| $K_2O$ | 4.60 | — | — | 2.81 | 1.56 |
| $P_2O_5$ | — | — | — | 0.01 | 0.30 |
| Sn | ++ | + | ++ | +++ | + |
| Mo | — | — | — | — | + |
| W | — | — | — | — | — |
| Pb | ++ | ++ | ++ | ++ | — |
| Cu | | | | | |
| $H_2O_{-110^\circ}$ | | | | 0.24 | 0.40 |
| $H_2O_{+110^\circ}$ | 0.47 | | | — | 0.81 |
| Total | 99.83 | — | — | 99.01 | 100.47 |

* I, II—Granites of the rapids of the Mras-su River (Porozhinski granites) (V. A. Moleva, analyst); III—granite of Mt. Klamar (V. A. Moleva, analyst); IV—granite of Mt. Elgen-Tag (N. Dedova, analyst); V—quartz keratophyre of Mt. Agabas (V. A. Moleva, analyst).

Note. + denotes traces of lines; ++, very weak lines; +++, weak spectral lines; ++++, spectral lines of medium intensity;—denotes the absence of the element.

is absent. (4) The titanium oxide content in these granitoids varies within a range of 0.25 to 0.70 per cent, attaining a considerable value in the granitoids of the Tey deposit (eastern slope of the Kuznetsk Alatau).

The petrographic structure of the regions here considered is characterized by the presence of intrusives composed by rocks of the gabbro-peridotite formation: Seglebir, Andabé (Kondoma region).

A petrochemical characteristic closely similar to the one given above is peculiar of the granites of another, more northerly group—the Tashelga group, from which the Porozhinski granites (biotitic and hornblendic) and the granites of Mt. Klamar* have been subjected to study. On the other hand, it should be noted that

* The spectroscopic determinations were made at the Laboratory for Spectrum Analysis, under the guidance of Prof. S. A. Borovick; the chemical analyses, at the Laboratory of the Petrographical Institute of the Academy of Sciences (1932—1937).

the granites of the Pustag Range, which play a considerable part in the Sheregesh and Shalym iron-ore deposits, are distinguished by a chemical characteristic of «pure granite type», without any traces of the influence of contamination phenomena of the granite magma with basic intrusives.

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*CHIMIE APPLIQUÉE*

# AUGMENTATION DU POIDS DE CHARGE DU CHARBON PAR ADDITION MICROSCOPIQUE DES HYDROCARBURES

Par A. AGROSKINE

*(Présenté par N. P. Tchijevski, de l'Académie, le 14. IV. 1945)*

La question de l'augmentation du poids de charge du charbon comme facteur fondamental d'une bonne production des fours à coke fait depuis longtemps l'objet des recherches de beaucoup de specialistes tout en Union soviétique qu'à l'étranger. La méthode d'augmentation du poids de charge du charbon présentée ici se distingue de toutes les méthodes connues par le fait qu'elle est basée sur le principe d'une action physico-chimique à la surface des particles de charbon*.

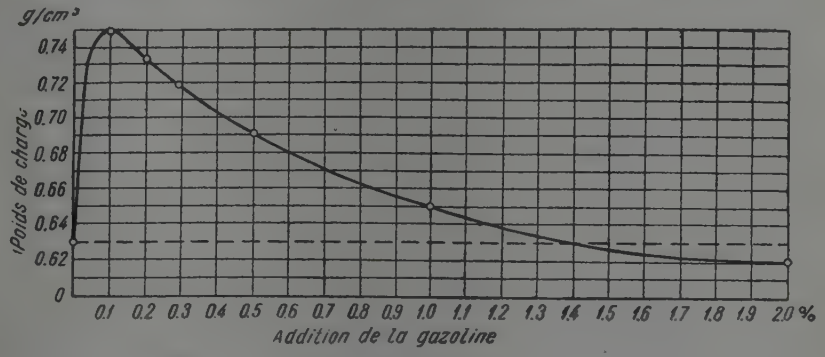


Fig. 1

Lors de l'addition à la poussière de charbon humide de hydrocarbures liquides, il se produit un phénomène très important pour la fabrication du coke: le poids de charge du charbon augmente considérablement (Fig. 1). On peut expliquer ce phénomène en supposant que l'hydrocarbure prend la place de l'eau à la surface des particules de charbon. La pellicule d'adsorption de hydrocarbure forme un lubrifiant entre les particules de charbon et empêche leur réunion en agrégats. On peut se représenter aussi un autre mécanisme de mouillage selectif suivant lequel, lors de l'addition des hydrocarbures, l'eau ne quitte pas la surface des particules de charbon, mais se couvre d'une pellicule d'hydrocarbure.

* Nous avons appris, après que la méthode décrite ici a été mise au point et appliquée dans plusieurs usines à coke de l'Union soviétique, qu'un travail à peu près analogue avait été entrepris aux Etats-Unis.

L'addition de matières douées de l'action de surface rend hydrophile la surface du charbon et diminue la grandeur absolue de son mouillage par des liquides non polaires. L'addition des phénoles, *i*-alcool amylique, acide oléique, *etc.*, abaisse l'effectivité du traitement de la charge de charbon par les hydrocarbures.

Le recouvrement de la surface du charbon par une fine pellicule de pyrolusite amène son hydrophilisation partielle, et comme résultat le poids de charge de la masse humide de charbon augmente de 1.1 à 3.9 pour cent.

La lyophilisation de la surface des particules du charbon par la paraffine augmente l'effet du traitement par les hydrocarbures. Simultanément, l'addition de paraffine augmente l'addition nécessaire optimum.

Pour rechercher l'explication du mécanisme de mouillage lors de l'addition dans la masse humide de charbon des hydrocarbures. on a determiné les angles de raccordement dans les systèmes: charbon-eau-air, charbon-eau-gazoline, charbon-eau-xylène, avec interversion dans l'ordre d'addition des liquides.

Par des expériences «de volume», c'est-à-dire en plongeant une mince tranche de charbon dans le liquide étudié, on a établi qu'une hystérésis bien prononcée a lieu au cours du mouillage. Donc la présence de ce phénomène dans les conditions de la production industrielle rend impossible le déplacement de l'eau par des liquides organiques.

Des porosités et des fissures microscopiques, la présence de différentes inclusions minérales et de composants pierreux font que la surface du charbon est de nature différente et devient hydrophobe ou bien hydrophile suivant l'ordre dans lequel les liquides sont ajoutés.

L'étude des processus pelliculaires, aussi bien que le calcul du «coefficient d'extension» montrent que le pétrole et aussi le benzène, le toluène et le xylène forment de fines pellicules sur la surface de l'eau.

La susceptibilité de la charge d'être mouillée par de différents liquides a été étudiée en détail par voie de détermination de la cinétique d'imprégnation dans de petites colonnes d'expérience de charbon concassé.

La cinétique de l'imprégnation du charbon concassé augmente avec l'accroissement de la polydispersion. Avec l'augmentation de la densité de la masse de charbon ayant une même dispersion, la cinétique d'imprégnation diminue. La présence d'une mince couche liquide à la surface des granules de charbon abaisse la cinétique d'imprégnation du charbon par le pétrole. Dans un charbon oxygéné la cinétique d'imprégnation est aussi abaissée par la présence d'une pellicule d'oxygène qui rend le charbon moins susceptible d'être mouillé par des liquides organiques non polaires.

Pour une masse de charbon d'une même composition et d'un même concassage, ce sont les hydrocarbures aromatiques non polaires (benzène, xylène) qui donnent les plus grandes vitesses d'imprégnation, puis suivent les produits légers de la distillation du pétrole (benzine, ligroïne, gasoline) et enfin les liquides oxygénés.

Ce sont les charbons dans le stade moyen de carbonisation qui se mouillent le mieux. La vitesse d'imprégnation est abaissée pour les charbons de formation plus jeune. L'imprégnation diminue à mesure que l'on passe de houilles grasses à des houilles maigres.

Dans le but d'étudier théoriquement la méthode décrite, on a procédé à la détermination de l'angle de pente naturelle de la charge de charbon $\alpha$. On a établi la dépendance de tg $\alpha$ de la hauteur et de la vitesse de chute du charbon et aussi son changement avec l'introduction d'eau et de gasoline dans la masse de charbon.

Le coefficient de friction, qui est défini par l'angle de pente, influence considérablement le poids de la charge, et dans de nombreux cas en est un facteur déterminant. L'angle de pente croît continuellement à mesure que l'humidité du charbon augmente jusqu'à 14 pour cent. L'absence de courbes absolument similaires de $\gamma$ et tg $\alpha$ indique que le poids de la charge dépend non seulement du coefficient de friction, mais aussi d'autres facteurs.

La grandeur de l'angle de pente est étroitement liée à la présence d'enveloppes d'adsorption et de solvatisation sur les particules de charbon. Dans le cas d'addi-

tion optimum des hydrocarbures. tg $\alpha$ prend approximativement sa moindre valeur (Fig. 2), ce qui indique une considérable diminution du coefficient de friction.

Ces expériences ont montré que l'humidité de la charge a une importance extraordinaire non seulement sur la grandeur de son poids, mais aussi sur l'accroissement de celuci-ci en cas de traitement par des liquides organiques. De plus, le degré d'humidité de la masse de charbon a aussi sa répercussion sur la quantité de liquide organique nécessaire pour obtenir le poids de charge maximum; c'est-à-dire que l'humidité détermine la quantité de liquide organique dépensée au mouillage du charbon (Fig. 3).

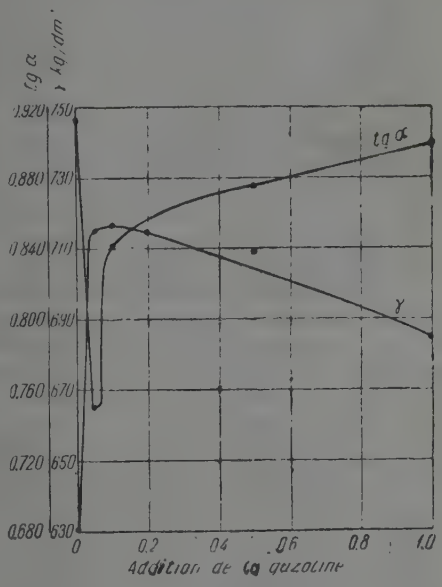


Fig. 2. $\gamma$ et tg $\alpha$ des masses de charbon en dépendance de l'addition de pétrole

Dans le processus d'oxydation, la surface des particules de charbon se couvre d'une mince couche d'oxygène, ce qui l'hydrophilise à un certain degré. Cela a comme résultat de diminuer l'angle de raccordement en cas de mouillage par des hydrocarbures.

La grandeur absolue du poids de la charge mouillée par la gasoline augmente continuellement à mesure que l'on prolonge le temps d'oxydation, parce que l'augmentation du poids due à l'oxydation d'une masse de charbon non mouillée couvre largement la diminution de l'efficacité du mouillage.

En cas d'oxydation partielle des hydrocarbures polymères qui entrent dans la composition de la houille, l'efficacité du mouillage par la gasoline diminue dans une proportion beaucoup plus considérable que l'efficacité du mouillage par huiles anthracéniques.

Les travailleurs de l'Institut Énergétique de l'Académie des Sciences de l'URSS et les travailleurs d'autres Instituts de recherches scientifiques et des usines ont

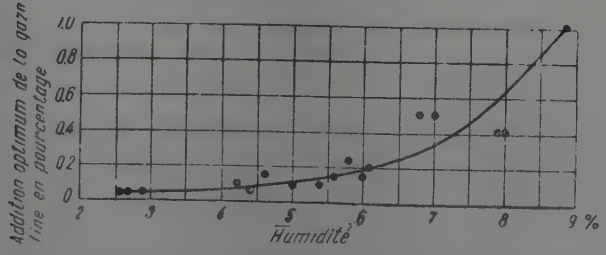


Fig. 3. Indices des quantités optimum d'addition de pétrole en dépendance de l'humidité des masses de charbon (pourcentage en poids)

pour la première fois mis en pratique la méthode décrite ci-dessus aux cokeries de Kemerovo, en janvier 1942; en mai de la même année, cette méthode fut appliquée industriellement aux cokeries de Nijni-Taguil. Dans ces deux usines on a utilisé la gasoline pour le mouillage dans une proportion d'environ 0.1 pour cent du poids de la charge.

Dans le but de trouver un produit de remplacement du pétrole et aussi pour mettre au point la méthode théorique, toute une série d'expériences ont été faites systématiquement au cours des deux dernières années. On a procédé au mouillage des masses de charbon par l'addition de différents goudrons, de produits chimiques forestières, de matières obtenues dans la production du semicoke, *etc.* Le meilleur résultat a été obtenu par un mélange de gasoline avec de l'huile anthracénique, qui a donné une plus grande augmentation du poids de charge que la simple addition de gasoline. A partir de 1943, les cokeries de Nijni-Taguil et de Kemerovo emploient principalement pour le mouillage des masses de charbon l'huile anthracénique pure ou mélangée avec d'autres huiles. A partir d'avril 1943 la méthode ENIN (Institut Énergétique) a été mise en pratique pour la production du coke au Combinat métallurgique de Magnitogorsk et en novembre1943 dans les cokeries du Combinat métallurgique de Kouznetsk, où on emploie aussi l'huile anthracénique pour le mouillage.

Les expériences pratiques et la longue exploitation industrielle de la méthode ENIN (Agroskine) ont permis d'établir ce qui suit:

a) L'augmentation de la production des cokeries en général n'est pas moins de 5 pour cent si la charge est mouillée avec de la gasoline et pas moins de 4 pour cent si l'on emploie de l'huile anthracénique. Pour certains produits (les plus importants—toluène, par exemple) l'augmentation de la production est beaucoup plus grande, le mouillage de la masse de charbon agissant favorablement sur la qualité du benzène cru.

b) L'augmentation du poids de la charge dans les fours influence favorablement la qualité des goudrons en diminuant le cracking. Dans ce cas, la teneur des goudrons en phénolocrésols et en bases pyridiques augmente. Leur poids spécifique diminue.

c) La qualité du coke métallurgique venant de masses de charbon mouillé est plus grande que la qualité du coke venant de charbons non mouillés.

d) Cette méthode peut être mise en pratique dans n'importe quelle cokerie, très rapidement. Elle peut être employée avec toutes sortes de charbons et différents degrés d'humidité de la charge.

e) La mise en pratique de cette méthode facilite considérablement le travail dans les fours à coke en simplifiant les opérations de chargement. D'autre part, la masse de charbon ne se congèle pas en hiver.

De nouvelles applications physico-chimiques pour le traitement du charbon sont envisagées, en particulier:

1. Emploi pour le mouillage d'un mélange d'huile anthracénique et de pétrole, ce qui augmente beaucoup le poids de la charge.

2. Addition dans la masse de charbon d'une petite quantité d'hydrophiles solides (par exemple, CaO) et de leurs mélanges avec des hydrocarbures.

3. Mise en pratique de la méthode ENIN dans les grandes fabriques d'enrichissement, ce qui apporterait une solution au problème de la lutte contre la congélation des charbons lavés.

4. Augmentation des sortes de charbon pouvant être employées à la fabrication du coke et, en premier lieu, l'utilisation du charbon à gaz.

Les travaux indiqués ci-dessus ont été executés, sous la direction de l'auteur, par l'Institut Énergétique de l'Académie des Sciences de l'URSS, avec la collaboration de l'Institut du charbon de Kharkov, de l'Institut de chimie metallurgique de la filiale de l'Académie des Sciences de l'URSS en Sibérie. Les bases théoriques de la méthode ont été établies avec la collaboration de P. Rehbinder, membre-correspondant de l'Académie des Sciences de l'URSS. De précieux conseils nous ont été donnés par A. Tchernitchev, membre-correspondant de l'Académie des Sciences de l'URSS.

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*GEOLOGY*

# THE TYPES OF THE CRETACEOUS SECTIONS OF FERGHANA

By O. S. VIALOV

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The Cretaceous sections within Ferghana do not remain constant. Their variation is so considerable that it would be difficult to give a description of the Cretaceous deposits as a whole. The sections of North Ferghana (Naryn type) stand out with particular clearness among the rest of sections. They cannot be correlated with any other section. Characteristic of South Ferghana is the Isphara type of sections while the Kizyl-Yar type is characteristic of North-eastern Ferghana, in South-eastern Ferghana a peculiar, Gulcha type is traceable. Some points of distinction enable us to separate out the Tashravat subtype which is a modification of the Isphara type in the south-western part of this region and the Kurama type developed in North-western Ferghana and comprised in the area of development of the sections of the Naryn type. All the types listed above are connected by intermediate sections, save for the Naryn type which stands apart from the rest.

In a previous paper the author considered the structure of the two principal types—the Isphara and the Kizyl-Yar type (3). He suggested a scheme of their subdivision and correlation.

In what follows are described variations of sections in other parts of Ferghana, and the characters of the remaining types.

## Variations in the Section West of Isphara

The sections of South-western Ferghana (Tashravat subtype) are distinguished from the Isphara type by the absence of most of the series described for the latter and by a resulting considerable reduction in thickness. The tendency towards a gradual disappearance westward of increasingly high horizons of the Cretaceous seems to be due to the marginal position of the given part of the basin. At Tashravat the Palaeozoic is directly overlain by a bed of gypsum with numerous fragments and thin plates of Palaeozoic shales. Its thickness is 2.5 m.

Higher up are developed red-coloured clays with green spots, not infrequently arenaceous, with a large quantity of rounded gypsum concretions up to 30 cm across, with bands of green sandstones and finely pebbly conglomerates, passing into coarse sandstone. The total thickness of this red-coloured areno-argillaceous member is no more than 12 m.

The Cretaceous section is completed by a member of white gypsum (Goznau series), 19 m thick.

Thus, in addition to the upper member of the Cretaceous section—the Goznau gypsum—only the member underlying it, of insignificant thickness, is present

here. It is impossible to correlate it with any definite series of the Isphara section. It may only be presumed to correspond to the whole Upper Cretaceous series of Isphara, from the Kan (or *Exogyra*) series to the variegated series inclusive.

## Variations in the Section East of Isphara

Proceeding east of Isphara, the following more or less considerable variations in the structure of the section may be noted.

On the Sokh River, instead of the thin band with sparse oysters, which in the Isphara section indicated the presence of analogues of the *Exogyra* series, a thick oyster bed is developed with *Ostrea oxiana* Rom. The oyster bed is overlain by a thick bed of massive limestone with innumerable natural casts of pelecypods and sparse sea urchins. The bed of limestone in the roof of the *Exogyra* layers (Muyan horizon) is very persistent and traceable far eastward (Kan, Muyan, Abshir, Yalpak-tash, Kachkar-ata).

The further changes in the eastward direction consist in that in the Lyakan series in the roof appears first a bed of gypsum while the thickness of the Lyakan limestones diminishes; then these rocks disappear entirely, and the whole Changet series, uniting into a single red-coloured formation, cannot be any longer subdivided into the three series distinguished in the Isphara section. The Kan series underlying the *Exogyra* beds east of Kan loses its normal habit, the conglomerates almost disappear and are substituted by sandstones and clays, and gypsum beds make their appearance.

The *Exogyra* series gradually becomes more clearly pronounced, its thickness increases, and green clay beds become prevalent within it; these are overcrammed with various oysters and contain ammonites as well (*Placenticeras*, *etc.*). Another constant bed of limestone traceable at the base of the *Exogyra* series is the Gulcha horizon (Kan, Muyan, Alshir, Yalpak-tash, Kachkar-ata, Gulcha). At last it becomes difficult not only to subdivide the variegated series, but even to separate it from the Yalovach series.

The distinctive features mentioned above caracterize the Gulcha type of sections, developed in South-eastern Ferghana. As a matter for regret the Cretaceous deposits of this particular region, which are of great importance for the solution of a number of general problems, have been the least studied, and scarcely any sufficiently detailed descriptions of these sections are available in literature.

## North Ferghana (Naryn type)

The Cretaceous deposits stretching in Northern Ferghana in the form of a belt along the base of Palaeozoic elevations are composed of red-coloured and pink sandstones and conglomerates. In each separate section they may be subdivided into several horizons, but it has not been possible as yet to extent them and to give a general scheme of the division of the sections of this (Naryn) type.

Let us give here a schematic description of the section along the Naryn River, based chiefly on data by G. I. Shatov, and on the author's cursory observations and some supplementary data furnished by M. G. Barkovskaya.

The upper non coal-bearing series of Jurassic grey-green sandstones is overlain by:

1. Brick-red clays with thin bands of brownish and greenish clays and argillaceous sandstones. The question as to the reference of these beds to the Cretaceous or the Jurassic has not been settled. Thickness, 70 m.

2. Pink conglomerates with a large quantity of lenticular gravel and sandstone layers up to 0.7 m thick. The presence of these layers gives these deposits a banded appearance. Thickness, 36 m.

3. Black analcime basalt. In literature this bed has been described as black sandstone. Its true nature was indicated for the first time in a paper by O. S. Vialov, S. F. Mashkovtzev and G. I. Shatov(2). This is the only place in Ferghana where Mesozoic igneous rocks are known. Thickness, 3.8 m.

4. Pink conglomerates with sandstone bands (analogous to layer 2). Thickness, 130 m.

5. A member consisting of clayey sandstones and highly arenaceous clays with bands of light-grey and pink fine-pebbly conglomerates, gravel beds and coarse sandstones; these bands project in crests on the slope, being distinguished both in the relief and by their colour. After the pale pink shades of the preceding member the bright red colour of the whole member is quite conspicuous.

In the upper part of the series is observable a decrease in the coarseness of the clastic material and the appearance of bands of cloddy light-pink and whitish clayey limestones and dolomites.

6. Pink coarse loosely cemented sandstones, interbedded with finely pebbly conglomerates. Amidst them are bands of red sandstones and numerous layers (up to 3.5 m thick) of cloddy red or mottled white-pink carbonate rocks, dissected by thin calcite veins. In some cases these are almost pure cloddy dolomites, limestones or marls with sparse sandstones; in other cases, highly arenaceous limestones or rather even very strongly calcareous sandstones; in other still it is even difficult to speak of the nature of the main rock, so dissected it is by minute calcite veins. These rocks do not generally form any continuous regular bed, but are interlocked with thin lenses of red sandy clays with numerous small quartz pebbles. They are frequently as it were intermixed with the clays enveloping them. Thickness, 400 m.

The total thickness of the Naryn section is 1240 m. It is not possible to correlate it with the sections of Southern and Eastern Ferghana.

## North-western Ferghana

The sections of North-western Ferghana, united as mentioned above in a separate Kurama subtype and still poorly investigated will not be discussed here at any length. The Cretaceous beds are known only at a few points and compose quite insignificant areas. The section consists almost exclusively of red-coloured rocks and its thickness is small. A sharp decrease in thickness is one of the features distinguishing it from the sections of the Naryn type proper. In some of the beds the fauna permits of certain correlations. In this respect the section at vil. Shaidan (on the meridian of Mt. Supé-tau) is of particular interest. Here resting almost directly on the Palaeozoic (separated from it by a 2 m thick soddy reddish band) are white and light-grey sandstones, now almost pure, now marlaceous, sometimes passing into cloddy marls. They are overcrammed with natural casts of pelecypods and gastropods, among which large *Acteonella* sp. are distinguished. Some fragments and one whole specimen of oyster, referring to the species *Exogyra turkestanensis* Born. have also been found. Thus it may be assumed that at Shaidan the Cretaceous section begins with *Exogyra* beds, and the entire Changet series is absent here. It is highly probable that this inference is to be extended to the other Kurama sections.

Higher up follows a member composed of red sands and clays with bands of pink and pinkish-grey sands, as well as rather sparse thin bands of denser grey and reddish sandstones and gravel beds, projecting in the form of ridges. In the talus a fragment of fossil wood has been found. The total thickness is 120—130 m.

The section ends with white and yellowish sands divided into two parts by a member (about 6 m thick) of interbedding red and light-grey greenish arenaceous clays passing into clayey sandstones. The total thickness is 20 m.

Higher up follows the Bukhara limestone. It is possible that the white sands, too, refer to the Bukhara stage. In such case the red-coloured formation corresponds to the Yalovach, variegated and Goznau series of Southern Ferghana.

The section near the Kansaï lead-zinc mine is also of interest.

Here are developed mainly red and pink banded loose sandstones (about 15 m thick); in the middle part of the deposits there is distinguished a horizon of gravel beds with a large quantity of bones of vertebrates and sections of pelecypod shells (osseous horizon).

It should be remembered, by the way, that at Isphara the osseous horizon occurs in the roof of the Yalovach sands.

The red-coloured member is overlain by an irregularly banded member composed of mottled light-grey and reddish arenaceous clays and argillaceous sandstones, sometimes marlaceous. Overhanging them is a cornice formed by layers of white marlaceous sandstones, in the lower part with two bands of mottled red and grey clays. In the middle part there is distinguished a layer (0.50 m) of white marly shell limestone with small natural casts of pelecypods and gastropods.

The latter layer doubtless represents the Bukhara stage.

The question as to the age of the separate series listed above will not be considered here. The data available at present are quite insufficient for settling this question. Indeed, the entire section contains only two series well characterized faunistically—the *Exogyra* series and the radiolite horizon—the fauna of which has not as yet been subjected to detailed examination. The possible age of the separate members of the section has been discussed in a previous paper[3].

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*GEOLOGY*

# PRINCIPAL TYPES OF CHEMOGENOUS $CaCO_3$ IN CARBONATE SEDIMENTARY ROCKS

By G. I. TEODOROVICH

*(Communicated by F. P. Savarensky, Member of the Academy, 5. II. 1945)*

While studying the carbonate rocks of the Palaeozoic of the Ural-Volga area and of the two slopes of the South Urals, and of the Mesozoic of the eastern slope, the author's attention was attracted by the multiplicity of modes of occurrence of chemogenous $CaCO_3$. Among these sedimentary carbonate rocks it is possible to distinguish, accordingly to their genesis, several types of chemogenous $CaCO_3$.

1. It is well-known that the upper horizons of large areas of recent oceans and certain seas, except for the cold regions and cold currents, are saturated with $CaCO_3$[9]. According to Wattenberg[10], the surface layers of the recent oceans are often oversaturated with $CaCO_3$, which is especially well pronounced in the warm waters of the tropical and subtropical belts. Oversaturation with calcium carbonate is associated with the high temperature[1] of the superficial layers of the warm belts of oceans [4] and of the shallow-water parts of warm seas, as well as with the life-activity of several vegetable organisms[4], which bring about an increase in the pH value [1] mainly by absorbing free $CO_2$. The oversaturation of water with $CaCO_3$, its mobility and the presence of mineral suspension of calcium carbonate and of some other substances (and colloids) result in that $CaCO_3$ is precipitated in the form of micrograins ($\leqslant 0.01$ mm); the minute size of $CaCO_3$ grains is also related to the low value of the ratio between the solid and liquid phase, or S : L of the system[3]. Such conditions are especially illustrative for the shallow-water portions of warm seas. On the contrary, the deeper layers of the ocean water and of the deeper portions of open seas are generally colder [4] and less alkaline[4]; part of the micrograined $CaCO_3$ formed in the upper portions of the water, and calcareous skeletons of planktonic organisms gradually sink, and part of them must be dissolved. In many warm-water shallow seas the distance between the zone of precipitation of micrograined $CaCO_3$ and the bottom of the basin is reduced, and in the lower horizons of waters no appreciable solution of $CaCO_3$ usually takes place.

The surface layer of marine bottom silts is usually characterized by the lowest pH values as compared to those of the bottom water and of the deeper portions of sediments [8,2]. The lowest pH values in the surface layer of silt are due to an intensive decomposition of organic matter, accompanied with the evolution of $CO_2$, of fatty and other acids, and also to the fact that the access of decomposition products to the water mass of the basin is impeded there. The cumulative influence of these two factors often results in the formation of a sharply acidulated interlayer in the silt, usually coinciding with the zone of transition from the oxidized part of the silt to its reduced horizons [7]. The higher values of pH in the lower layers of the marine silt are associated with a lower content of organic matter and the extinction of oxidation processes, mainly with a decrease in the $CO_2$ production,

associated with the decay of organic matter ([8],[5]). Besides, the processes of decomposition of organic matter largely take place in the upper layers of the silt([6],[2]). Along with the accumulation of calcareous silt its gradual dehydration takes place; the pH of the silt waters usually becomes somewhat higher (as compared to the surface silt layer). There arise exchange reactions between the waters and the enclosing deposits, especially in case products of decomposition of organic matter are present, and as the deposits are gradually sinking to a greater depth, the temperature and the mineralization of the water are raised. A slow precipitation of $CaCO_3$ is thus made possible. The calcium carbonate additionally produced here does not form new grains, but will serve for the accretion of the previously existing grains of the sediment.This will take place because precipitation of $CaCO_3$ in the overlain sediments usually proceeds in case the water is motionless, the ratio of S : L of the system ([3]) is high, the oversaturation of silt with $CaCO_3$ but slight or at any rate if the degree of oversaturation changes gradually. Thus are formed conditions necessary for a solidification of the micrograined calcareous material in overlain sediments. The higher the rate at which conditions for a slow precipitation of $CaCO_3$ are created in the process of diagenesis, the more rapid is the lithification. The latter will take place at a much higher rate if the bottom sediments are dried up, being raised above the water level. The specifically low degree of cementation of writing chalk is apparently due to the unfavourable conditions for slow additional precipitation of $CaCO_3$ in the overlain sediment.

2. Another, sharply different type of chemogenous $CaCO_3$ of sedimentary carbonate rocks is represented by c a l c a r e o u s c r u s t s, in particular by the characteristic incrustations; in all these cases $CaCO_3$ is deposited directly upon the solidified or solid bottom of the basin, making its surface even and sometimes filling up the cavities of the upper part of the solidifying sediments. The formation of $CaCO_3$ crusts at the bottom of the basin could have taken place at a period when the conditions prevailing on the surface of the bottom were favourable for a direct precipitation of $CaCO_3$, *i. e.* when the sea-bottom water possessed a higher temperature or pH values ([1]) closely approaching those of its surface-layer. These conditions are usually available in very shallow-water portions of warm seas, incrustation being especially characteristic of reefs. In some rare cases crusts are formed also at greater depths, under conditions of constant warm bottom currents. The rare occurrence of fragments of incrustations in reefs is evidence of the high rate of solidification of the latter, and of their being formed largely within relatively closed areas of the elevated parts of the reef.

3. The data available on calcareous öolites show that they were formed in very shallow-water portions of warm seas and partly in lagoons characterized by water mobility, and that the deposition of öolitic concentres usually took place around solid particles, chiefly in a suspended condition and partly when these were rolled over the bottom. Under these conditions and because of the high mobility of waters, the surface of the bottom cannot be covered with crusts, since the sediment itself is mobile, and the transported particles are dragged over its surface.

4. It is necessary to discriminate between s y n g e n e t i c and e p i g e n e t i c recrystallization. The conditions of syngenetic recrystallization are best pronounced in reefs. The author has found that the area of incrustation in reef limestones is usually* separated from the areas of micrograined $CaCO_3$ by a diffuse zone of recrystallized $CaCO_3$, becoming gradually coarser towards the area of incrustation. The $CaCO_3$ of this zone was formed under conditions of near-bottom and silt waters transitionary between the possibility of passive accumulation of micrograined $CaCO_3$ on the bottom surface and the optimum for the formation of calcareous crusts on the bottom.

On the surface of calcareous silt and within its topmost portions there takes place an intense decomposition of organic matter, accompanied with a considerable

* Cases of incrustation $CaCO_3$ directly adjoining micrograined one have been observed at the surface of fissures only in the case where the micrograined deposits had previously emerged from under the water level and become cracked.

evolution of $CO_2$, which permits of a partial dissolving of the $CaCO_3$ of the silt and of its passage into the silt solution. When the sediment is overlain by new portions, the decomposition is extinguished, which can usually be seen from the higher values of the pH of the silt water in the deeper horizons of the sediments, as compared to its surface layer. Thus in the overlain silt necessary conditions are created for a slow oversaturation of the silt water with $CaCO_3$ and for the accretion of the previously formed indivisible particles of the micrograined sediment. Along with the increase in the pH, favourable influence also produced the rise of temperature which may accompany the sinking of the sediment to a greater depth at later stages of diagenesis corresponding to cases of slowed down lithification with recrystallization within the fine grain ($\leqslant 0.01$ mm). Recrystallization may be stimulated by conditions of shallowing, if these bring about the rise of temperature in the near-bottom and silt-water, or an increase in the pH. In all these cases recrystallization proceeds more intensely within the upper part of the calcareous sediment, which results in a rapid lithification of the latter.

Syngenetic recrystallization mostly affects only a part of the limestones. A change in the degree of recrystallization of limestones according to the bed reflects the changes taking place in the near-bottom silt conditions. Typically enough, in limestones which have been rather intensely recrystallized syngenetically, well-preserved cemented material is usually observed.

5. Crystallized $CaCO_3$, usually calcite, coating the walls of the voids, can be either of syngenetic or epigenetic origin. Syngenetic calcite of this type is formed inside the semisolidified or solidified calcareous sediment, most frequently in its deeper horizons. Thus in the Upper Palaeozoic reefs the scalenohedral calcite, coating the walls of cavities, is frequently post-incrustational. Epigenetic crystallized calcite is clearly secondary, forming on the walls of rock cavities; it is precipitated from circulating waters or associated with processes of dedolomitization.

6. Coarse-grained irregularly isometric $CaCO_3$, with irregular outlines, is, as a rule, clearly secondary, being formed by way of epigenetic recrystallization, or filling up rock cavities; in some cases it fills voids at late periods of the diagenesis of the sediment. Unlike syngenetic recrystallization, epigenetic recrystallization can destroy the cementing material, too. In the case of dedolomitization, the coarse-grained $CaCO_3$ usually formed shows irregular baroque lobate outlines and not infrequently, inclusions of residual fine-grained dolomite.

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*HYDROGEOLOGY*

# ON THE HYDROGEOLOGICAL CLASSIFICATION OF GEOSTRUCTURAL ELEMENTS

By N. K. IGNATOVICH

*(Communicated by F. P. Savarensky, Member of the Academy, 8. VI. 1945)*

1. Changes that take place in hydrogeological conditions are governed mostly by definite laws of a regional nature, and may be traced both over vast territories representing large geological and structural elements, and over limited areas, such as a separate horizon, complex, part of a structural or geomorphological element. Changes in the chemical composition of waters, their regime, resources, *etc.* also proceed in a definite sequence which may be expressed in terms of hydrogeodynamic and hydrogeochemical zoning (1).

2. Within vast territories the zoning is borne out by the geological and structural peculiarities of the territory. The development of the structural elements of the latter furnishes the most important data as to the stages of the development and formation of underground waters. No subdivision of a territory into its hydrogeological regions can be done unless it is subjected to structural analysis. According to the author's views, a hydrogeological region is a part of the territory within a geological structure or a separate element of the latter, which is characterized by similar features of the occurrence of waters and a similar type of the hydrogeological processes which are responsible for the regime (dynamics), resources and chemistry of the underground waters.

3. In order to form a better idea of the regional hydrogeological conditions, the author recommends to supplement the existing geological classifications of structural elements, determining their development and age relations, to classify the structural elements (complexes, horizons).

A. According to the degree of d i s c l o s u r e, distinguishing (a) disclosed, (b) partly disclosed, and (c) closed structures or elements.

B. According to the degree of d r a i n a g e: (a) drained ones, (b) partly drained or slightly drained, (c) undrained.

C. According to the degree of w a s h i n g: (a) washed, (b) partly washed or those in process of washing, and (c) unwashed, in process of salinification.

4. A d i s c l o s e d structure is characterized by a considerable extent of erosion, an open and hypsometrically high (usually above sea level) position of the complexes of rocks, series, horizons. Unlike this, a closed structure is characterized by an open occurrence in younger deposits only. The principal complexes occur generally at a considerable depth below sea level.

A typical example of disclosed structures is furnished by crystalline massifs (Baltic shield, Ukrainian massif, *etc.*), axial zones of mountainous folded areas (the Urals, the Caucasus, *etc.*). If a large geological and structural region be analysed with regard to the complexes of the rocks developed, the disclosed structures are most frequently found to be composed of the oldest rocks.

,From the hydrogeological standpoint, the disclosed structures are characterized by the development of a zone of active water exchange. Within the area of these structures which are for a long time subject to weathering, fresh waters of considerable dynamic resources are mostly formed. The static reserves of the waters of these structural elements are usually restricted.

5. A typical example of closed structures is offered by tectonic depressions, *e. g.* the Caspian depression, the inner parts of the near-Moscow syneclise, *etc.* From the hydrogeological standpoint they are characterized in the first place by a zone of impeded circulation; and at greater depths, by conditions of a stagnant water regime (regime of relative quiescence). The closed structures are most frequently fed by way of migration of underground waters from outside, from other, better disclosed structures. In closed structures waters with an increased or a high mineralization are most frequently formed, attaining at deeper horizons saturation with salts of chlorine-sodium-calcium brines.

6. Partly disclosed (semiclosed) structures embrace a very large group of peripheral parts of large tectonic downwarpings, depressions or slopes of anticlinal bulges, uplifts. For instance, the Moscow artesian basin, the Dnieper-Donetz depression may be considered as semidisclosed structures having independent catchment areas; and as such into which the more open structures are drained. These structures are usually characterized by three hydrogeodynamic zones (zone of active water exchange, zone of impeded circulation and zone of stagnant water regime) to which corresponds a definite geochemical nature of underground waters.

7. The degree of d r a i n a g e of the structures or their separate elements depends on the existence of a regionally formed underground drainage, which generally proceeds from structurally and hypsometrically elevated regions towards structural depressions and sea or oceanic depressions.

If the structures are viewed in the aspect of geological time, they all must be regarded as being more or less drained. The degree of drainage is practically different within the same structure in different complexes of rocks. Sometimes the underlying deposits are drained to a higher degree than the overlying ones. Besides, the degree of drainage of the structures varies in time.

8. Epeirogenic oscillations cause changes in the distribution of the catchment areas and underground discharge; the tectonic disturbances cause a migration of underground waters along predominant directions; on the other hand, the ancient paths of circulation of underground waters are often gradually obstructed due to a precipitation of hydrochemical sediments along fissures. A conclusion can be made that in the zones of Alpine folding and fracturing the drainage of the structures and complexes of rocks extends to greater depths and is more intense than in the zones of Hercynian or some other, older folding. With regard to the degree of washing, however, the relations may be reverse.

9. In general, the open structures are the most drained, while the closed are the least drained ones. Among the closed structures some separate elements or complexes may be more or less drained, too. For instance, the Terek-Kuma depression which is part of the vast Caspian depression, or the near-Azov-Kuban depression, forming the south-eastern termination of the near-Black Sea depression include artesian basins of abundant fresh water. These basins should be considered as drained ones in certain parts of the structures, supplied, like many piedmont depressions, by underground waters from the northern slope of the Caucasus.

10. The degree of drainage of the structures or of their elements causes under definite conditions their w a s h i n g; it stands to reason, however, that a drained structure is not a synonym of a washed structure, deprived of any readily soluble salt.

In some cases, in spite of a considerable degree of drainage, even a gradual salinization of certain parts of the structure takes place.—a phenomenon widespread in arid regions in the case of superficial deposits; salinization takes place also as a result of decomposition of halogene sediments and the migration of salts along with waters to new reservoirs. A s e c o n d a r y s a t u r a t i o n of rocks with salt is a phenomenon as widespread as their freshening, desalinization.

11. The most washed structures are disclosed drained ones, especially in case they are composed of many crystalline and metamorphic rocks. Under platform conditions the higher lying complexes have generally been to a considerable extent washed, while the deeper lying deposits enclose brines, representing unwashed complexes.

A typical example of such partly washed structures is offered by the Moscow artesian basin (the Carboniferous has been washed, the Upper Devonian is being washed, the Middle Devonian encloses brines of high concentration), and by the Dnieper-Donetz artesian basin (the Meso-Cenozoic has largely been washed while the Palaeozoic has not).

12. Closed, slightly drained structures are very slowly washed, and the changes that result are displayed and fixed on a geologic time scale. But under definite conditions these structures (*e. g.* in piedmont areas, by virtue of the peculiarities in the dynamics of underground waters migrating from the mountains to the zones of depressions) become at places drained in certain directions. Owing to such a drainage, selective washing of rocks is produced, which results in the fact that under mineralized waters fresh waters may frequently occur without any marked water-resistant horizon.

13. A semidisclosed drained structure may also be an unwashed one. For instance, the structure of Cheleken island (Caspian Sea) which has numerous ascending mineral waters is an unwashed one. The freshening of this structure may possibly take place at a subsequent stage of its geological history; within this structure, young hydrocarbonate and alkaline waters, moving from catchment areas (Kopet-Dag), displace the ancient chlorine-calcium waters, and along certain zones of disturbances find their way to the surface completing a long circular path of water on the surface.

14. In analysing the palaeogeographical conditions and tectonic peculiarities of a given territory, it is necessary to take into account the part played by the hydrogeological agencies. Intensely drained structures, washed during a geologically prolonged time, may lose their readily soluble salts either partly or completely.

For instance, the Carboniferous deposits within the Moscow artesian basin, *i. e.* in the sphere of circulating fresh waters, are devoid of gypsum, while outside this basin, where mineralized waters and brines occur in the Carboniferous deposits, gypsum is rather widespread, often showing evidence of a primary occurrence. It may be assumed that the absence of gypsum in the Carboniferous of the near-Moscow region is of a secondary nature. After the zone of active water exchange and considerable washing of rocks has extended northward and north-eastward, further inside the near-Moscow syneclise, the zone of leached sulphates from the carbonate Carboniferous deposits will be correspondingly shifted to new areas.

15. Within the Russian Platform numerous strictly local disturbances are known which, though of considerable intensity, often defy correlation with the general picture of the platform dislocations of the given territory. Mention should be made, for instance, of the Puchezh dislocations in the Gorki region, where amidst disturbed variegated Tatarian deposits, huge blocks of limestones of Kazan age are encountered.

It may be supposed that here the Kazan deposits had been squeezed out to the surface by a stock of Lower Permian salt which later on was destroyed by the work of underground waters.

The occurrence of Mesozoic and Quaternary rocks in the areas of salt springs in the Yaroslavl province, very low from the hypsometrical standpoint but confined to a very restricted area, should be most probably due to the sinking of these rocks to a destroyed salt-dome structure.

16. The zone of development of salt-dome structures appears to extend beyond the boundaries of the Caspian depression. Many areas of development of halogen facies of the Permian, Devonian and at some places, possibly, of the Lower Palaeozoic, too, on the Russian Platform, under conditions of a sufficient depth of occurrence, might have been the zones of formation of salt-dome structures.

From geological data it may be presumed that the intensity and number of salt dome structures beyond the boundaries of the Caspian depression must be less considerable. Moreover, outside the Caspian depression, as shown above, the degree of disclosure drainage and washing is higher, which affects unfavourably the conditions necessary for the preservation of salt domes.

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*CYTOLOGY*

# DISTRIBUTION OF RIBONUCLEIC ACID IN THE CYTOPLASM AND NUCLEI OF DIFFERENT CELLS

By G. ROSKIN

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Cytologists have of late taken great interest in the occurrence and distribution of cytoplasmic nucleic acid (which is also called yeast nucleic, or ribonucleic), as well as in the rôle it plays in the life of the cell. The lack of accuracy involved in the methods of cytochemical analysis, as for example, the use of different solvents of pepsin or trypsin in experiments on digestion, *etc.*, and finally the staining with different dyes,—all this leads to controversial or insufficiently significant results.

An important achievement in the study of ribonucleic acid has been made by Caspersson [6] who suggested a new sensitive cytochemical method for the analysis of a number of cell proteins. It consisted in studying spectra of absorption of ultra-violet rays by definite cell-components and in comparing these spectrograms with the spectrograms previously obtained for a number of definite proteins, *viz.* thymonucleic acid, ribonucleic acid, histons, globulins. However, Caspersson's method requires the use of a rather complicate equipment, which somewhat restrains its range of application. For determining ribonucleic acid to be recommended is Brachet's method, which is based upon the treatment of cells either in sections or smears, with the enzyme ribonuclease, and upon a subsequent comparison of the preparations obtained with the control ones [3]. This method has been described in detail in a previous paper[9]. The present note is concerned with a number of questions bearing upon the distribution of ribonucleic acid in various cells.

1. Changes suffered by ribonucleic acid in the course of the life-cycle of protozoic cells. We have already succeeded in demonstrating [8–10] that the plasm of the various *Protozoa—Infusoria, Coccidia, Trypanosomata, Plasmodium vivax, Amoebae*—is exceedingly rich in ribonucleic acid. It may be assumed that the presence of this acid is responsible for the comparative basophily of the plasm of a protozoic cell as compared, say, with most cells of mammalia, which have a clearly expressed acidophilic plasm. Thus, according to the content of ribonucleic acid, the cells of *Protozoa* may well be compared with embryonic or undifferentiated cells, which are also high in ribonucleic acid [see data by Brachet [4,5]]. The idea suggests itself as to a possible direct or indirect relation existing between the presence of ribonucleic acid in protozoan and embryonic cells, on the one hand, and the intense processes of division and growth, on the other. In my opinion the existence of such a relation is quite probable.

While studying the content of ribonucleic acid at different moments of the life-cycle of a protozoan cell, one soon becomes aware of the fact that it varies both quantitatively and with respect to the mode of distribution in the different parts

of the cell. Thus it is found to be sometimes diffused all over the plasm, or concentrated at the poles of the cell only, accumulated at the periphery of the cell or perinuclearly, *etc.* The regularities underlying these variations in the distribution of ribonucleic acid stand in need of further investigation. The only thing that seems quite obvious is that the amount and distribution of ribonucleic acid is liable to change under the influence of certain external agencies with much greater ease than this has been reported for thymonucleic acid. E. A. Shubnikova, who against my suggestion had been studying the influence of different agents upon the content of ribonucleic acid in *Paramoecia*, succeeded in establishing that relative starvation caused an appreciable decrease in the content of ribonucleic acid which set in 12 hours after the beginning of the experiment. In the process of conjugation the quantity of nucleic acid undergoes regular changes. As a rule, the conjugating couples of *Paramoecia* do not differ appreciably as to the content of ribonucleic acid, which remains stable during the early period of conjugation. It is in late ex-conjugates that alterations set in, consisting in the breakdown of the old macronucleus and the resorption of the nucleoproteids, on the one hand, and in the formation of a new nuclear apparatus and the resulting synthesis of nuclear proteids (thymonucleic acid), on the other hand. A considerable increase in the content of ribonucleic acid in the plasm of the *Paramoecia* can be observed at this period.

At the time being it is difficult to decide what is the source at the expense of which this process is going on. Two assumptions may be advanced: 1) the increase in the content of ribonucleic acid proceeds at the expense of the synthetic activity of the protoplasm, this acid being formed as an intermediate product necessary for the synthesis of nucleoproteids; 2) the amount of ribonucleic acid increases at the expense of the rearrangement of the nucleoproteids of the old nuclear apparatus which undergoes decomposition.

In either case we have to bear in mind that the mechanism of conjugation processes is not restrained merely to nuclear processes as it was assumed hitherto: the relations between the thymonucleic acid of the nucleus and the cytoplasmic ribonucleic acid seem to play no minor part in the complicated process of conjugation and restoration of the nuclear apparatus.

2. Ribonucleic acid in the plasm of nerve-cells. The discussion of the chemical constitution of Nissl bodies (the «tigroid») gave rise to a comprehensive literature on the subject. The assumption hasarded by some of the authors as the start of the discussion, *viz.* that the composition of the «tigroid bodies» involves nucleoproteids, seemed to have suffered an utter defeat after it had been clearly demonstrated with the aid of Feulgen's reaction that Nissl bodies were destitute of thymonucleic acid. This debatable question seemed to have ultimately found its solution. As a matter of fact, Brachet([4],[5]) showed by means of his method that the «tigroid bodies» did actually contain ribonucleic acid. This discovery was checked by the present author on ganglion cells of the spinal cord of rabbits. On the author's sections the basophilic component of the «tigroid» was clearly seen to vanish after the preparation had been treated with ribonuclease, but there was another proteinic component of acidophilic nature, which still remained after this treatment. The results obtained could thus be interpreted to mean that there were at least two protein components in the composition of the «tigroid», and one of them was the ribonucleic acid.

It is well known that nerve cells of invertebrates are, as a rule, destitute of Nissl bodies. Of special interest appeared therefore the observation made at this laboratory by L. Lewinson and Ionova on the nerve cells of the leech and of *Anodonta*. Lewinson and Ionova have been successful in showing that the plasm of these nerve cells is high in ribonucleic acid. It may consequently be assumed that nerve cells of invertebrates do also contain Nissl substance which is not accumulated in them into definite clots, but remains distributed in a diffused manner throughout the protoplasm of the nerve cell. The transition of this substance to a form typical of the «tigroid bodies» seems to be associated with a definite degree of development of the nervous system.

3. Ribonucleic acid as a component of the nucleus. It was taken for granted during a rather long period of time that the so-called cytoplasmic nucleic acid (ribonucleic acid) occurred in the plasm of the cell only, unlike thymonucleic acid which was regarded as an indispensable component of the nuclear substance. The observations reported below make us, however, revise this conception.

1. The karyosome of the nuclei of *Amoebae* of the «limax» type contains ribonucleic acid as one of its components.

2. The macronuclei of the infusoria *Paramoecium* and *Euplotes* also contain rather large amounts of ribonucleic acid.

3. The nuclei of *Trypanosomata* (*Trypanosoma equiperdum*) are likely to contain some ribonucleic acid, distributed in a diffused manner.

4. The nuclei of oocytes of rabbit *Coccidiae* contain ribonucleic acid, impregnating their nucleoli.

5. The nucleoli of nerve cells of *Anodonta* contain ribonucleic acid as one of their components.

6. As shown by observations carried out by E. A. Shubnikova, the large nucleoli of Balbiani's giant nuclei of the salivary glands of *Chironomus* contain a certain quantity of ribonucleic acid.

7. With the aid of Brachet's method it is possible clearly to bear out the presence of a considerable amount of ribonucleic acid in the nucleoli of nuclei of muscular and cartilaginous cells of the axolotl.

8. The nucleoli of embryonic cells of a chick embryo show a well-pronounced positive reaction for ribonucleic acid.

9. Large nucleoli of the nerve cells of the spinal cord of rabbits comprise two proteinic components at least, one of which is ribonucleic acid.

10. The nuclei of the various tissues of the human organism contain ribonucleic acid which is concentrated in the nucleoli of their nuclei.

We are still unable to decide whether ribonucleic acid is or is not an indispensable component of the nucleolus. The observations reported above show, however, that yeast nucleic acid is of frequent occurrence in the nuclei of the most various cells, where it enters the composition of the nucleoli as one of their components.

The present author has paid special attention to the nuclei of cells of malignant growth in man, which he investigated in much detail. And he could not help feeling amazed by the considerable quantity of ribonucleic acid that is found to be accumulated in the hyperthrophied nucleoli of these cells. Thus, along with a large amount of cytoplasmic nucleic acid ([10]), which is found to be distributed diffusely throughout the plasm of the cells of malignant growths,—a feature they have in common with embryonic cells,—these cells contain also a strikingly high amount of this acid concentrated in their nuclei. But this is no specific character inherent in cancerous cells, as it might be assumed at the first sight. Observations carried on by the present author on nuclei of gland cells of the mammary gland of mice during the period of pregnancy have shown that these cells, when subjected to functional stimulation, are likewise increased in volume and contain increased amounts of ribonucleic acid, although in a somewhat smaller quantity than nuclei of cells of adenocarcinome of the mammary gland of mice.

A definite relation may thus be assumed to exist between the amount of ribonucleic acid and the intensity of vital processes. As one of the working hypotheses accounting for the rôle played by the ribonucleic acid we are to adopt the view advanced by Caspersson, which may be formulated in the following way: at any place there is synthesis of proteins, nucleic acids are also present. In the plasm we find ribonucleic acid, in the nucleus, thymonucleic acid. An important correction should, however, be introduced, *viz.*: ribonucleic acid is also an essential component of the nucleus.

Caspersson's hypothesis, even with the correction stipulated above, fails, however, to embrace all the possible functional importance ribonucleic acid may assume in the life of the cell. The presence of ribonucleic acid cannot be associated merely with protein synthesis, as may be seen from the occurrence of this acid in the «tigroid

bodies» of the nerve cell. It may be supposed that the part played by the ribonucleic acid, or by its derivatives in the life of the cell is greatly manifold.

All that has been said above points to the necessity to approach the old problem as to the functional rôle of the nucleolus at a different angle. According to the hypothesis which has been thoroughly developed by Heidenhain (1909) and a number of subsequent authors, among whom Maximov (1918) should be cited, the nucleolus is an inert product of metabolism, an accumulation of ballast proteins, destitute of nucleic acid, which the cell rejects during karyokinesis.

A different hypothesis had been advanced by Flemming (1882) and subsequently developed by various students among whom were also Bogoyavlensky (1911) and Dogiel (1925). According to this hypothesis, the nucleolus is intimately connected with the process of chromatin regulation in the nucleus, being a transformer of the chromatin substance of the latter. The observations discussed by us in this note speak in favour of the latter hypothesis, especially if we take into account the reasons advanced by Brachet ([12]) in favour of the fact that ribonucleic acid serves as a source for the production of thymonucleic acid.

Our observations, moreover, have revealed that the behaviour of thymonucleic and ribonucleic acids varies according to the period of the life cycle of the nuclei of cells of *Metazoa*: during interkinesis thymonucleic acid is as a rule dispersed in the general mass of the nucleus, while ribonucleic acid is concentrated in a constant organoid, the nucleolus. During mitosis an inverse phenomenon is observed, thymonucleic acid being concentrated in the chromosomes, whereas the substance of the nucleolus is as a rule in a dispersed state. More complicated processes involved in the behaviour of the ribonucleic component in gland cells, as well as in the ripening of sex cells, will be the subject of our subsequent investigations.

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*GENETICS*

# ON THE NATURE OF CROSSING-OVER INDUCED BY X-RAYS IN MALES OF *DROSOPHILA MELANOGASTER*

By N. I. SHAPIRO

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The problem on the nature of the exchange of homologous chromosome segments taking place in males of *Drosophila melanogaster* under the influence of X-rays cannot be regarded as solved. First of all, the mechanism of the action of X-rays upon crossing-over in males is not clear. Two solutions may be suggested.

1. The action of X-rays causing crossing-over in males is identical with that responsible for chromosome aberrations. Thus viewed the induced crossing-over is a sort of aberration, the cross-over chromosome being merely a special case of aberrant chromosomes.

2. The mechanism of the action of X-rays producing crossing-over in males is qualitatively different from that operative in the production of chromosome aberrations. The induced exchange of homologous segments is of precisely the same nature as in the case of crossing-over normally taking place in females. The action of X-rays is limited merely to creating conditions which favour the realization of exchange (for instance, of a more close conjugation of the homologous chromosomes).

The former hypothesis may provisorily be termed «the aberration hypothesis», the latter may be named «the crossing-over hypothesis». There are number of reasons in favour of either of them. The following facts are in favour of the crossing-over hypothesis: a) The exchange takes place at homologous loci ([2],[7]); b) the exchange takes place at the four-strand stage ([2]); c) the cross-over chromosomes are mostly non-lethal in the homozygous condition ([2],[7]).

In favour of the «aberration hypothesis» speak certain *a priori* considerations which may be put forward in the first place. One of the basic results of exposure to X-rays upon the chromosomes is the breakage of the chromosome. According to numerous studies, chromosome breakage takes place in the cells of any tissue of any organism so far investigated. The fate of the broken-off parts may be different. Some of the fragments are lost in the very first cell division: some breakages are restored by re-union of the fragments at the point of breakage; some fragments are united in a new sequence, giving rise to the various chromosome aberrations. A great many chromosome aberrations are formed in the sperms of *D. melanogaster* under the influence of X-rays. However, when cells are X-rayed at the early stages of gametogenesis the rate of occurrence of chromosome aberrations is sharply reduced, practically falling to zero ([10]). At the same time, approximately at the same stages, the exchange of homologous segments begins to occur. It can hardly be supposed that at early stages of gametogenesis X-rays would cease to cause chromosome breakage. The suggestion is more plausible that the destiny of chromosome breaks in these cells is different from that in sperms. When one assumes, in accordance with the majority of students ([13], [9], [4]) that the chromosome breaks pre-

cede re-union and are independent of it, and that the mode of re-union depends on a number of agents, in the first line, on the mutual position of the points of breakage and on the possibility of the movement of the broken-off parts ([6],[4]), one may easily imagine that the breakage taking place when the chromosomes are in a state of sufficiently close conjugation at the four-strand stage, would lead either to the re-union of the segments in their original arrangement, or to the exchange of homologous segments (in the case of a simultaneous breakage of the two sister-strands at identical loci).

Considered from this point of view, crossing-over caused by X-rays in males is nothing else than an aberration process taking place under specific conditions of chromosome conjugation. It may be inferred from the above said that the basic arguments in favour of the crossing-over hypothesis (exchange at homologous loci at the four-strand stage) are not against the aberration hypothesis either. Along with *a priori* considerations some observational data can be presented in support of the aberration hypothesis. Thus, it has been demonstrated that the distribution of breaks in the male approaches more closely the one which is typical of translocations, than in crossing-over in females([2]). Some support for the aberration hypothesis is furnished also by the fact that a certain fraction of the cross-over chromosomes have a recessive lethal effect inseparable from the point of breakage ([2],[7]). It appears to be fairly evident, therefore, that the question on the nature of the exchange of homologous chromosome parts in males or, to be more exact, on the mechanism of the action of X-rays upon this process, should be regarded open.

One of the possible means to study the mechanism of the action of X-rays upon the crossing-over process is the study of the relation of this process to the dose of radiation. A number of different types of relation between the rate of chromosome aberrations and the dose of X-rays are known at present. According to data obtained from studies of *Tradescantia* ([11]), there exists a direct proportionality of the X-ray dose in the case of the one-hit aberrations (*i. e.* aberrations resulting from a single ionization, or from several but simultaneously acting ionizations), whereas in the case of two-hit aberrations (*i. e.* such as result from two independent ionizations, or two groups or ionizations), the frequency of aberrations is proportional to the square of the dose. When sperms of *Drosophila melanogaster* are X-rayed, the two-hit aberrations (translocations, inversions) exhibit an intermediate relation: with the dose increasing two times the rate of aberrations increases three times. This peculiar behaviour is accounted for by the specific conditions of union of the broken-off parts ([12], [4]).

When immature sex cells of males of *Drosophila melanogaster* are X-rayed and the crossing-over in males is of aberrational nature, one should expect a situation similar to that discovered in *Tradescantia*. If the breakage of the two non-sister chromatids is effected by one «hit», a direct proportionality to the X-ray dose should be expected. If, on the other hand, two independent «hits» were necessary, the rate of exchange would be proportional to the square of the dose.

In order to find the relation between the exchange of homologous regions and the X-ray dose, one-day-old heterozygous males $\frac{b\ cn\ c}{+\ +\ +}$ were given the following doses: 1500 r ($t_1$), 3000 r ($t_2$) and 4500 r ($t_3$)*. After treatment the males were individually placed with virgin females and left with them during seven days. The progeny of these females was discarded. After seven days the males were again individually placed with virgin females b cn c and left with them for 4 days (1st brood). After another 4 days the males were transferred to new virgin females b cn c (2nd brood), *etc.* There were several such transfers in each experimental set. The results are summarized in the table.

It may be observed from the table that the rate of exchange increased rapidly with the transition from the smallest dose to the medium, while there was but a

* The experiment consisted of 5 sets. Irradiation was done with the aid of different apparatus. It is possible that the doses administered were not always exactly 1500, 3000 and 4500 r, but they approached these doses and, at any rate, one may be sure that within each set the relative doses were strictly 1 : 2 : 3. This relation was ensured by the time of exposure.

slight increase when passing from the medium dose to the maximum. A sort of logarithmic type curve is observed as one may see from the accompanying figure. In plotting the curves use was made of the percentage of crossing-over (total for the regions b—cn and cn—c), deduced from the five experimental sets.

Progeny from Crossing ♂ $\frac{b\ cn\ c}{+\ +\ +}$ × ♀ $\frac{b\ cn\ c}{b\ cn\ c}$

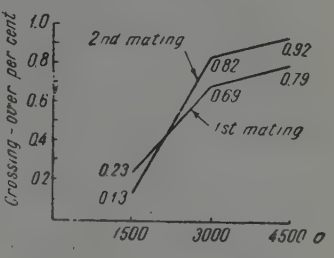
| Set No. | Brood No. | Dose of X-rays | Number of flies examined | Number of cross-over flies (total for the regions b—cn and cn—c) | Crossing-over, per cent |
|---|---|---|---|---|---|
| 1 | 1 | $t_1$ | 927 | 2 (2)* | 0.21 (0.21) |
| | 1 | $t_2$ | 585 | 1 (1) | 0.17 (0.17) |
| | 1 | $t_3$ | 209 | 1 (1) | 0.48 (0.48) |
| | 2 | $t_1$ | 231 | — | 0.00 |
| | 2 | $t_2$ | 451 | 3 (2) | 0.66 (0.44) |
| | 2 | $t_3$ | 273 | 1 (1) | 0.37 (0.37) |
| 2 | 1 | $t_1$ | 678 | — | 0.00 |
| | 1 | $t_2$ | 733 | 2 (2) | 0.27 (0.27) |
| | 1 | $t_3$ | 443 | 1 (1) | 0.23 (0.23) |
| | 2 | $t_1$ | 538 | — | 0.00 |
| | 2 | $t_2$ | 872 | 7 (4) | 0.80 (0.46) |
| | 2 | $t_3$ | 1727 | 8 (7) | 0.46 (0.40) |
| 3 | 1 | $t_1$ | 698 | — | 0.00 |
| | 1 | $t_2$ | 1792 | 7 (7) | 0.39 (0.39) |
| | 1 | $t_3$ | 1801 | 7 (8)** | 0.44 (0.44) |
| | 2 | $t_1$ | 645 | 1 (1) | 0.15 (0.15) |
| | 2 | $t_2$ | 1494 | 8 (8) | 0.53 (0.53) |
| | 2 | $t_3$ | 1639 | 20 (10) | 1.22 (0.61) |
| 4*** | 2 | $t_1$ | 921 | 2 (2) | 0.22 (0.22) |
| | 2 | $t_2$ | 953 | 7 (7) | 0.73 (0.73) |
| | 2 | $t_3$ | 675 | 4 (4) | 0.60 (0.60) |
| 5**** | 1 | $t_1$ | 408 | 3 (2) | 0.73 (0.49) |
| | 1 | $t_2$ | 458 | 5 (4)***** | 1.94 (1.55) |
| | 1 | $t_3$ | 150 | 3 (2) | 2.00 (1.33) |
| | 2 | $t_1$ | 1438 | 4 (4) | 0.28 (0.28) |
| | 2 | $t_2$ | 1004 | 14 (9) | 1.39 (0.90) |
| | 2 | $t_3$ | 2108 | 41 (7) | 1.94 (0.33) |
| | 3 | $t_1$ | 824 | — | 0.00 |
| | 3 | $t_2$ | 1478 | 10 (4) | 0.68 (0.27) |
| | 3 | $t_3$ | 932 | 2 (1) | 0.21 (0.11) |
| | 4 | $t_1$ | 612 | 2 (2) | 0.33 (0.33) |
| | 4 | $t_2$ | 879 | 23 (3) | 2.62 (0.34) |
| | 4 | $t_3$ | 1203 | 7 (4) | 0.58 (0.33) |

* In parentheses are indicated the numbers of independent occurrences of crossing-over.
** Double crossing-over was recorded as a result of two independent breaks.
*** In the 4th series owing to accident only the second mating could be analysed.
**** In the 5th series there were analysed four four-day-long matings.
***** Additional cross-over classes were recorded as one occurrence.

The curves obtained do not coincide with what had to be expected according to the aberration hypothesis. Moreover, the mode of relation between the rate of exchange of homologous chromosome parts and the irradiation intensity in males is similar to the dependence of the rate of crossing-over on the X-ray dose in females. As shown by Belgovsky (1), the variation in crossing-over due to X-ray dose also follows an logarithmic type curve. Such a similarity of the curves seem to be an argument in favour of the crossing-over hypothesis. It seems to me that such a conclusion would be poorly founded, or at any rate premature. One of the possible explanations of the relation observed consists in the assumption that the rate of crossing-over observed is the result of the action of X-rays upon two qualitatively different and independent processes. On the one hand, the X-rays act upon the process of crossing-over, *i. e.* upon the frequency of chromosome breaks; on the other, they cause death of the immature sex cells. The killing of the cells by X-rays takes place in a selective way in the sense that cells possess different sensitivity at different stages of gametogenesis. A number of facts are in favour of the suggestion of selective effect, among them the phenomenon of temporary sterilization of the X-rayed individuals observed by many students (2, 3, 5, 8) and found also in our experiments. In particular there is ground to believe that late spermatogonia and early spermatocytes are more susceptible than young spermatogonia (2). It has been demonstrated, moreover, that crossing-over is easier effected at these stages than at the early spermatogonia stage(2). Thus an increase in the rate of chromosome breaks takes place

along with the increase in the dose of X-rays and, at the same time, is reduced the number of cells in which crossing-over may be called forth by the influence of X-rays. Thus to be expected is just the sort of relation found in our study. The explanation put forward by us is applicable on principle also to data on the effect of different X-ray doses upon the crossing-over in the female.

One may suppose here also that along with the increase in X-ray dose the number of cells in which crossing-over could be effected is reduced. Another possible explanation of the relation observed consists in the admission of the effect of the time factor as observed for two-hit aberrations by Sax [11]. Inasmuch as the adjustment of the intensity of irradiation was done in our work by changing the duration of exposure, it was possible that, along with the increase in dose, the probability of the re-union of the fragments in the original sequence was increased. Thus the logarithmic type of the curve illustrating the dependence of crossing-over upon the dose does not contradict the aberration hypothesis. Further experiments are necessary for the solution of the problem.

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*PLANT PHYSIOLOGY*

# RUBBER FORMATION AS AFFECTED BY THE GEOGRAPHICAL FACTOR

By P. F. MEDVEDEV

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The ability of certain plants to produce rubber has undoubtedly been developed under the influence of their specific life conditions and of the geographical medium, in the first place. To a certain extent this may be seen from the geographical distribution of the rubber-plants over the globe, their adaptation to definite natural areas.

The great majority of rubber-plant species (about 65 per cent) occur in the tropical zone of America, Asia and Africa. They are mainly trees, lianas and shrubs; only a few of them are perennial herbs.

In the temperature zone of America, Asia and Europe there occur about 30 per cent of the total of rubber-plants of the globe. They are localized here in torrid and arid regions (deserts and semi-deserts of America and Asia). Most rubber-plants of the temperate zone are herbacious perennials, more seldom, shrubs or half-shrubs, and very rarely, annuals. All rubber-bearing species of the temperate zone are clearly xerophytic. They do not occur as a rule in moist localities.

However, the various groups of rubber-plants are not affected in the same way by the geographical factor. According to their reaction to the geographic factor the rubber-plants of the temperate zone have been classed by us into two groups, the one comprising all the leaf- or resin-rubber-plants and the other all the root-rubber-plants.

## Leaf-rubber-plants

The representatives of this group show on intense reaction to the geographical factor by changing their rubber-producing capacity.

The best rubber-yielders of the genus *Asclepias*, *A. erosa* (containing up to 13 per cent of rubber in its leaves) and *A. subulata* (up to 6 per cent of rubber) are distinguished by their rubber-producing capacity in their native regions (half-deserts of Arisona and California). When transferred to different geographic surroundings, however, their capacity undergoes a strong change. Under the conditions of Kuban, where these species were tested by us within the period 1938—1942, there is not heat enough for their normal development. As a result, the accumulation of rubber proved to be a very low one, *viz.* 1.2—1.4 per cent in *A. erosa*, and 0.3—0.4 per cent in *A. subulata*.

*Parthenium* which is a subtropical shrub has found its new home in certain regions of Transcaucasia where it produces high yields of green mass and accumulates more rubber (8—10 per cent) than in its native land. When it is transferred to the temperate zone of the North Caucasus (Otrada Kubanskaja) where it does not survive the winter, rubber accumulation drops to 1.1—1.2 per cent.

A very interesting example is offered by a representative of the genus *Solidago*, *S. virgaurea*. It is a non-rubber producing plant, widely spread in northern latitudes, where the conditions do not favour rubber formation and where rubber is not a biological necessity for the plant. We have transferred this species into the zone of variable moisture in Kuban region where, as shown experimentally, the formation and accumulation of rubber is favoured by natural conditions. As a result we have obtained for the first time in a non-rubber producing species rubber accumulation attaining 3.6—4.2 per cent in leaves.

On the other hand, rubber-bearing species of the genera *Asclepias*, *Apocynum* and *Solidago*, when transferred to the northern latitudes, suffer a sharp drop in their rubber content. In the period of 1936—1938 we cultivated them with the special purpose to study rubber formation in relation to the geographical factor. The results obtained were very convincing. Here we are reproducing only a single table illustrating the behaviour of three species (belonging to different genera and families) grown at different geographical localities.

Table 1

Effect of Geographical Factor upon Formation of Rubber and Resins in Leaves of Rubber-plants

| Locality | *Asclepias syriaca* | | *Apocynum cannabinum* | | *Solidago serotina* | |
|---|---|---|---|---|---|---|
| | rubber content, in % | % of resins | rubber content, in % | % of resins | rubber content, in % | % of resins |
| Pushkin | 0.42 | 4.12 | 0.29 | 3.90 | 1.31 | 8.91 |
| Gorky | 0.95 | 4.45 | 0.30 | 4.38 | — | — |
| Kinel (Kuibyshev province) | 3.26 | 4.78 | 3.58 | 8.70 | 1.67 | 5.70 |
| Kuban province | 4.43 | 6.34 | 4.46 | 5.56 | 3.76 | 8.22 |

Not only the rubber content is increased at southern latitudes, but the amount of resins is reduced.

## Root-rubber-plants

Within the group of root-rubber-plants the geographical factor has a less pronounced effect upon rubber formation. Comparatively good results as to the rubber content have consequently been obtained in the northernmost areas of the crop, *viz.* in Leningrad, Gorky and Ivanovo provinces, where the leaf-rubber-plants fail to accumulate rubber. However, in this group, too, the same regularity is observed, namely, a better though less pronounced accumulation of rubber farther to the south.

The results of the very first geographical sowing of kok-saghyz, krym-saghyz and tau-saghyz made in 1933 went to corroborate this statement.

Literary data on the rubber content in kok-saghyz are summarized in Table 2 (averages for each region).

The figures reported by Yurgenko for tau-saghyz are very illustrative (2). The average rubber content in the one-year-old roots in Central Asia is 7.7 per cent, in Voronèzh, 6.7 per cent, and in Leningrad, 2.8 per cent.

Conclusions. 1. The hereditary capacity for rubber formation has been developed in plants of the temperate zone under the influence of the extremely severe living conditions (soil aridity, dry air, high temperature, all the three factors being very variable; periodicity of precipitations and shortage of the latter, intense insolation, *etc.*).

Table 2

Effect of Geographical Factor upon Rubber Content in Roots of One-year-old and Two-year-old Kok-saghyz

| Province | Rubber content in the roots, in % | |
|---|---|---|
| | first year | second year |
| Leningrad | 6.68 | — |
| Ivanovo | — | 7.43 |
| Moscow | 7.09 | 7.57 |
| Dniepropetrovsk | 7.31 | 8.83 |
| Krasnodar | 8.10 | 10.47 |
| Stavropol | 9.21 | — |

2. As the species is spread and differentiated into new species, the ability of rubber formation is levelled out under the influence of the new ecological and geographical conditions. However, when such a species is transferred to a medium favourable to rubber formation, the ability becomes fully manifest again.

3. Rubber-bearing species of leaf-rubber-plants, when transferred to northern localities, do considerably reduce the accumulation of rubber in their leaves; species producing little rubber when grown at lower latitudes do, on the contrary, noticeably increase their rubber production. The rubber production capacity of leaves is regularly increased along with the migration from north to south.

4. In root-rubber-plants the formation and accumulation of rubber is also dependent upon the geographical factor, although this relation is less pronounced than in the group of leaf-rubber-plants.

5. The rubber plants of the temperate zone may be divided into two following types, according to the degree to which they react for the geographical factor.

First type, plants which show an intense reaction for the change in the geographical medium. Rubber formation in this group is directly connected with the assimilatory activity of the cells. In this case rubber is the primary product of the synthesis of assimilates. The intensity of its accumulation depends therefore to a great measure on the conditions favouring the accumulation of assimilates. Here belong the so-called leaf- or resin-rubber-plants, *viz. Asclepias, Solidago, Parthenium, Apocynum, etc.*

No successful results can be obtained with these crops unless they are grown in sufficiently southerly localities, where the conditions are favourable for intense assimilatory activity.

To the second type are referred plants which show a less pronounced reaction for geographical location. In this group rubber formation is not directly connected with photosynthesis, for rubber is formed in the laticiferous vessels rather than in assimilatory tissues. Here rubber is evidently a secondary product of exchange. The hereditary capacity for rubber formation does not vary much under different natural conditions, unlike its behaviour in the plants of the first type. Into this group belong all our native root-rubber-plants groups, *viz.* kok-saghyz, krym-saghyz and tau-saghyz.

Better practical results will also be secured here when the crops are grown under conditions of adequate regional distribution, the relative and absolute output of rubber being duly taken into account.

6. Being chemical compounds formed by the plant under the influence of definite environmental conditions, rubber-substances play a definite biological rôle and participate in metabolic processes. Their formation is a procedure contributing to the conservation of the plastic material at the period when the conditions are especially unfavourable for the growth and development of the plant.

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*PLANT PHYSIOLOGY*

# SEXUAL DIMORPHISM IN DIOECIUS PLANTS

By V. N. NAUGOLNYKH

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The existence of morphological and physiological distinctions between the two sexes in dioecious plants and the possibility of the production according to environmental conditions of flowers of either sex both in dioecious and monoecious plants have been described in literature. The statement advanced by Joyet-Lavergne [1] on plasmic sexualization is thus in a certain measure supported. *Rumex acetosa* may serve as an example of strong sexual dimorphism. According to my observations made at the Troizk reserve (Cheliabinsk region), these differences are confined to the rate of development and to vigour. As a rule the blooming of the male individuals is somewhat delayed as compared to that in the female plants. The average height of the female plants is 74 cm, that of the male plants, 46 cm. Wild hemp is an example of reverse relations: here the male plants are in full bloom when the females are just developing flower-buds. In still another dioecious plant found in the same locality, *Melandrium album*, no morphological differences in the vegetative organs or in the rate of development are found. We have chosen this species for the study of certain physiological indices, namely, transpiration and respiration. Transpiration was determined with the aid of a torsion balance with a short exposure (2 minutes); in studying respiration Ivanov's method was used. The area of the leaves was measured by weighing imprints obtained on photographic paper. Prior to the determination of the area the leaves were saturated with water.

The experimental plants were taken from different habitats. *Melandrium album* was found in the steppe in groups consisting of 12—15 individuals; it could also be found as a weed in the cultivated fields. The habitus of the plants taken from these two habitats differed greatly: while the plants growing in the steppe were 40—45 cm high, with their stems branching only in its upper portion, near the inflorescence, and had rather thick pubescent leaves, the individuals found among crops attained 60—65 cm in height, their stems produced many lateral shoots at the base, thus forming an ample bush, bearing delicate, thin leaf-blades.

The intensity of transpiration is illustrated by Table 1.

In the steppe (exper. made on July, 28) the transpiration proceeded at a higher rate in male plants. This difference is especially manifest in the upper leaves. In the crop-field (experiment made on July, 21) where the plants of *Melandrium album* have a more mesophytic habit, the relations were reverse: the upper leaves of the female plants showed a considerably stronger transpiration as compared to the corresponding leaves of the males. It may thus be inferred that the female plants of *Melandrium* are of a more xerophytic type and possess greater adaptive possibilities. When placed under less favourable conditions, which is undoubtedly the case in the open steppe with its abundantly grass-covered soil, stronger insolation and better heating of the air, their transpiration is reduced. In the field with mellow

Table 1

| Story of leaf, counted from apex of stem | Transpiration intensity, mgs per dm² | | | |
|---|---|---|---|---|
| | July 21, oat crops | | July 28, steppe | |
| | ♂ | ♀ | ♂ | ♀ |
| 1 | 21.62 | — | 70.37 | 34.44 |
| 2 | 43.74 | 58.07 | 92.11 | 51.65 |
| 3 | 49.00 | 65,47 | 71.13 | 55.33 |
| 4 | 39.6 | 43.01 | 40.01 | 26.84 |
| 5 | — | 32.44 | — | — |

soil covered with rather high panicles of oats, where *Melandrium* constitutes the lower story, the humidity regime of the plants is more favourable so that the transpiration of the female plants proceeds with a higher energy than in the male ones. The comparison of the temperature regime in the steppe and in the crop-field supports this idea (Table 2).

Table 2

| Date | Maximum temperature at 1 o'clock p. m. | |
|---|---|---|
| | steppe | crop field |
| 17.VII | 32.6 | 28.7 |
| 20.VII | 26.7 | 24.2 |
| 27.VII | 35.2 | 29.0 |

Table 3

| Story of leaf counted from base | Amount of $CO_2$ in mgs per dm² pro 1 hour | |
|---|---|---|
| | ♂ | ♀ |
| 5 | 2.24 | 3.13 |
| 6 | 2.55 | 4.44 |

The determination of respiration in the leaves of *Melandrium album* has shown that gaseous exchange proceeds with greater intensity in the female plants than in the male ones (Table 3).

It should be noted that for determining the rate of respiration the leaves severed from the plants were placed into closed boxes containing some water and were brought to the laboratory. Experiments were therefore made on leaves saturated with water so that the difference in the rate of respiration could not be put down to the differences in the water content of the cells. Palladin(²) has noted the increase in the intensity of respiration of the vegetable organs after these have been kept in a solution of methylene blue. According to experiments by Bogoslovskaja made at this laboratory, which were confirmed by subsequent work, a treatment of the seeds of cucumber with a weak solution of methylene blue produced a considerable increase in the number of female flowers in certain varieties of cucumbers.

It seems highly probable that this shift toward a greater number of female flowers was due to a modification in the course of the oxidation-reduction processes under the influence of methylene blue. The difference in the rate of respiration in male and female *Melandrium* as observed by us is, therefore, of interest as a possible manifestation of plasmic sexualization.

Aside from these physiological studies, a morphological analysis of the flowers of *Melandrium* was made in order to test their homogeneity. Warming has pointed out (³) that *Melandrium* species possess traces of the floral organs of the opposite sex, while the perianth is identical in both sexes. As a matter of fact in the majority of male flowers there is a primordial pistil in the form of a small body widening towards its base and bearing a style of varying length. In a number of flowers there

are moreover abnormalities in the number of stamina, which varies from 7 to 11. In the female flowers containing normally 5 sepals, 5 petals and 5 styles, the number of these parts may vary or a transition of the styles into petal-like formations may take place: the styles become flattened, more translucent and remind petals. Table 4 contains a numerical survey of these deviations.

Table 4

Male flowers

| | Number of flowers |
|---|---|
| 5 sepals, 5 petals, 10 stamina | 6 |
| 5 sepals, 5 petals, 10 stamina, primordial pistils | 696 |
| Varying number of stamina (7—11) | 14 |
| Total | 716 |

Female flowers

| | |
|---|---|
| 5 sepals, 5 petals, 5 styles | 126 |
| Varying number of petals (4—6) | 3 |
| » » » sepals (6) | 1 |
| » » » styles (4- 6) | 9 |
| With petal-like styles (primordial doubleness) | 10 |
| Total | 149 |

A primordial pistil in the male flower is thus of common occurrence, and purely unisexual flowers occur but very rarely (0.8 per cent of cases). No stamina were found in the female flowers. In connexion with the data of Schopfer and Blümer[4] on flower-modification in this species taking place under the influence of fungal infection, the occurrence of such variation in the flower structure of *Melandrium album* points to a high plasticity of this species.

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*HELMINTHOLOGY*

# A CONTRIBUTION TO THE KNOWLEDGE OF THE DEVELOPMENT CYCLE OF THE LARVAE OF CESTODES PARASITIC OF THE LIVER OF THE REINDEER (*CERVUS TARANDUS*)

By E. P. PUSHMENKOV

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As well known, the various organs and tissues of the reindeer (liver, lungs, muscles of the heart and trunk) are often found to be infested by larvae of tapeworms, cysticerci of the genus *Taenia*, which attain their sex-maturity in the intestine of the dog. In the organism of the reindeer the cysticerci cause severe morbid processes which may occasionally be fatal for the animal invaded by them. For example, the experimental infestation of a reindeer-calf with two sex-mature segments of *Taenia* taken from a dog resulted in such severe cysticercosis of its liver, lungs and heart that the young animal succumbed towards the 15th day upon infestation.

Two species of cysticercoids parasitic of the reindeer are known in literature: *Cysticercus tenuicollis* (parasitic of the serous membranes), whose larval stage is the *Taenia hydatigena*, parasitic of dogs, and 2) *Cysticercus tarandi*, parasitic of the skeletal muscles, whose larval stage is the *Taenia krabbei* of the dog. Cysticerci from the liver, lungs and heart of the reindeer were usually referred to the species *C. tarandi*.

According to the frequency of occurrence, intensity of invasion and of their pathogenic effect, the foremost place is held by the cysticerci parasitic of the liver of the reindeer. These cysticerci represent comparatively large bladders, usually rounded in shape, ranging from 10 to 18 mm across. They are localized either in the body of the liver, or under its capsule. The cysticercus is surrounded from outside by a connective tissue membrane which envelops the bladder proper of the cysticercus. The latter is filled with a transparent liquor; the head and neck of the parasite are evaginated. The scolex is the shape of a small pea, yellowish in colour. The length of the head and neck attains 2.1 mm; the width, 1.56 mm; the diameter of the scolex is 0.84—1.01 mm. The scolex is armed by four suckers and a double crown of hooks situated upon the proboscis. The diameter of these suckers varies from 0.24 to 0.30 mm; the horizontal diameter of the crown bearing the hooks is 0.33 mm. The latter are 30 in number. The largest of them attain the size of 0.22 - 0.23 mm; the length of the grip is 0.11 mm; its maximum width, 0.029 mm; the length of the appendix, 0.031 mm. The small hooks are 0.13 - 0.15 mm long. The length of the grip is 0.058 mm, against a width of 0.017 mm; the appendix is 0.04—0.046 mm long.

The study of the cysticerci from the cardiac muscle of reindeers revealed their morphological identity with the cysticerci from the liver. Contrary to this, a comparison between the cysticerci parasitic of the liver and heart of the reindeer and those localized in its skeletal musculature has shown that they are to be classed with two distinct species.

In order to solve the question as to the specificity of cysticerci parasitic of the different organs of the reindeer, the present author undertook a series of experiments, involving the infestation of dogs with reindeer cysticerci. The results of these experiments are briefly described in the present communication.

Under experiment were three puppies, 10 days of age. Previous to infestation the puppies were suckled by their mother. One of the puppies was infested with 8 mature segments taken from the liver of a reindeer; another puppy was infested with 4 cysticerci taken from the hind limb of a reindeer. In both cases infestation was practised *per os*. The third puppy was left as control. After infestation the animals were kept under conditions entirely precluding any possibility of secondary invasion. Two months upon infestation, the three puppies were sacrificed. The autopsy of the dog infested with cysticerci from the liver of a reindeer showed its small intestine to contain 4 individuals of tapeworm. Several isolated mature segments of tapeworm were also discovered in the lumen of the intestine. The small intestine of the dog infested with cysticerci taken from the muscles of one of the extremities of a reindeer were found to contain three specimens of *Taenia*. In the lumen of the intestine individual mature segments of the worm were discovered. The intestine of the puppy left as control proved uninfested with cysticerci.

A. The cestodes discovered in the young dog infested with cysticerci from the skeletal muscles of a reindeer were found to be typical representatives of *Taenia krabbei*.

B. The cestodes found in the puppy infested with cysticerci obtained from the liver of a reindeer proved to be representatives of a new species which had not yet been described in literature. Its brief description is given below.

Delicate cestodes, white in colour; length of strobilae from 70 to 94 mm; segments 275—280 in number. Scolex armed; 4 suckers present, 0.33—0.34 mm across. Proboscis equipped with 30 chitinous hooks. The larger hooks attain the length of 0.23—0.24 mm, against a maximum width of 0.025 mm. The smaller hooks are 0.13—0.16 mm long. The horizontal diameter of the crown attains 0.52 mm; the head of the parasite—1.26 mm in diameter; the length of the neck attains 1.6 mm. Hermaphroditic segment 4—5 mm long and 7—9 mm wide. Maximum width of strobilae, 10—11 mm. The length of a mature segment is 11 mm, against a width of 6 mm. The angles formed by the segments are almost right; the posterior edge of each segment is somewhat broader than the anterior edge of the next segment. Sex apertures alternate irregularly and are situated in the medium part of the segment.

From what has been said above it should be inferred that the tapeworms which developed in the intestine of dogs fed with cysticerci from the liver of reindeer are representatives of an independent species for which we suggest the name of *Taenia parenchymatosa* nov. sp.

In order to decipher the developmental cycle of *Teania parenchymatosa* and to verify the assumption ventured above as to the taxonomic position of the cysticerci discovered in the liver of reindeer being that of an independent species, the following experiment was arranged.

A reindeer, a fortnight old, kept before the experiment and during it under conditions precluding any possibility of invasion, was infested *per os* with two mature segments of *Taenia parenchymatosa* obtained from one of the dogs used for the experiments described in the first part of this note. After a fortnight the reindeer succumbed. Upon autopsy ruptures of the liver and haemorrhages into the peritoneum were established. Both liver parenchyme and the blood discharged into the abdominal cavity were found to contain huge numbers of cysticerci which had the appearance of small bladders, 5—8 mm long and 3—4 mm across. The lungs and the cardiac muscle were also found to contain cysticerci 1 and 9 specimens, respectively. A thorough examination of the muscles of the trunk, as well as of the epiploon and of the other parenchymatous organs gave negative results.

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*HELMINTHOLOGY*

# A NEW *FILARIA* FROM THE ARTICULATION SOCKET OF BIRD TIBIA

By A. N. CHERTKOVA

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The present article contains a description of nematodes discovered in the articulation socket of the tibia of *Circus aeroginosus* in Armenia, near village Elenovka, on the shore of lake Sevan, on July 25th, 1930. The material, which consists of one male and five females has been kept at the Museum of the Veterinary Scientific Research Institute in Erevan, Armenia, up to the present.

As a result of thorough examination, the nematodes were found to be representatives of the family *Filariidae*; they were to be classed as a new species. *Pelecitus armenica* nov. spec., belonging to the genus *Pelecitus* Raill. et Henry, 1910.

The genus *Pelecitus* was described in 1910 (1) by Raillet and Henry, who made it embrace 5 *Filariidae* species parasitic of birds: 1) *Spiroptera helicina* Mol., 1859; 2) *P. circularis* Mol., 1859; 3) *P. tercostatus* Mol., 1859; 4) *P. serpentulus* Dies., 1851; 5) *P. quadripapillosus* Mol., 1859. As a matter for regret the founders of the genus *Pelecitus* did not suggest any precise generic diagnosis.

A detailed description of the species *P. tercostatus* was given by Skrjabin(4) in 1916. To the five species enumerated above this author added a sixth species, *viz. Filaria calamiformis* Schneider, 1867.

In 1926 Yorke and Maplestone(3) referred all the six species to the genus *Pelecitus* referring the latter to the family *Aproctinae*. In 1927 Walton described a 7th species of the *Pelecitus* genus, *viz. P. alatus*. And in 1932 Sprehn(2) referred to the genus *Pelecitus* its 8th species, *Filaria clava* Wedl., 1855.

On the basis of data collected by Skrjabin and Shikhobalova (unpublished), the former author classed with the genus *Pelecitus* also the species *Spiroptera helix* Linstow, 1899, which was the 9th one.

## *Pelecitus armenica* nov. sp.

Host—*Circus aeroginosus*: localization, the articulation socket of tibia; place of finding, Armenia, lake Sevan.

Description. A nematode of medium size, light-yellow in colour. Cuticle delicately shaded in transverse direction. Anterior and posterior body-ends rounded. Mouth devoid either of labra or chitinous ornamentations, opening terminally and leading into a cylindrical oesophagus, which forms no division. Cephalic end equipped with 12 submedian and 2 lateral papillae arranged in three concentrical sets. The internal set consists of six circumoral papillae, of which 2 are arranged laterally, 2, submedially-ventrally, and 2, submedially-dorsally. The medium set consists of four papillae arranged submedially. The external set is made up of four double papillae which are also distributed submedially.

M a l e. Body-length 9 mm, with a maximum width of the medium part of the body attaining 0.602 mm. Caudal end rounded, equipped with delicate symmetrical wings. The width of the latter at the level of the cloaca equals 0.021 mm. The cloaca is arranged 0.060 mm apart from the caudal end of the body. There are 7 pairs of papillae on the ventral surface of the caudal end. Of these 4 pairs are pre-anal, showing the shape of little mushrooms, and 3 pairs of post-anal ones, two of which are also mushroom-like in shape, while the smallest, third pair is the shape of a pedicel. Above the cloaca is situated an odd sessile papilla. The first pair of posterior, pre-anal papillae is situated close to the cloaca, the second pair, 0.018 mm apart from it; the second, the third and fourth pairs have equal space intervals between them (0.012 mm). The first pair of anterior, post-anal papillae is

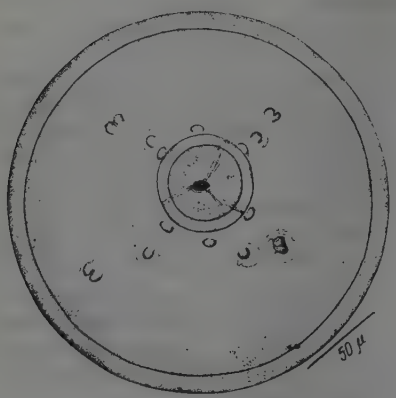


Fig. 1. *Pelecitus armenica* nov. sp. Sucker apparatus of the cephalic body end, female

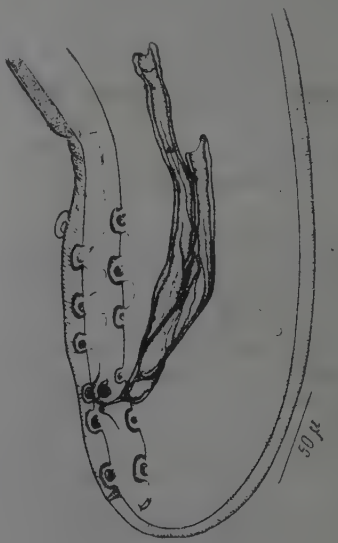


Fig. 2. *Pelecitus armenica* nov. sp. Caudal body end, male

also situated close to the cloaca; the second pair is separated from the first by a distance of 0.012 mm, while the third pair, which is the shape of small pedicels, is arranged 0.007 mm apart from the second pair. Body width within the region of the cloaca, 0.094 mm. Spicules small, uneven, very delicate, poorly chitinized, tubular-cylindrical in shape, with undulatory lateral edges. The spicules are distinguished from one another chiefly by the structure of their distal ends. The large spicule attains 0.150 mm in length, its proximal end is 0.012 mm wide, with a saddle-like depression, while the distal end is 0.008 mm wide, truncate. The smaller spicule attains 0.110 mm in length, against a width of the proximal end attaining 0.012 mm. Its proximal end shows a minute indentation and is somewhat cut aslant. The distal end terminates as a truncate digitate appendix, reaching 0.007 mm in length. Close to its transition to the constricted part the width of the spicule is 0.010 mm, while the width of the very end of the appendix is 0.001 mm.

F e m a l e. Body-length 15.5 mm, against a maximum width of the medium part of the body equal to 0.817 mm. At the level of the location of the vulve the width of the body attains 0.320 mm, while at the level of the end of the oesophagus it is 0.400 mm. The oesophagus is 0.935 mm long. The anterior part of the oesophagus at the level of the nerve ring is the narrowest, reaching no more than 0.025 mm. At the level of its transition to the intestine the oesophagus attains its maximum width 0.064 mm. The nerve ring is 0.160 mm apart from the cephalic end; the

caudal end is rounded, bearing one pair of small, slightly pronounced caudal papillae, arranged on the sides of the anus, which is situated terminally, or nearly so. The vulve is arranged at the level of the posterior third of the length of the oesophagus, 0.560 mm apart from the cephalic end; the labra of vulve somewhat protruding.

Differential diagnosis. The new species here described, *Pelecitus armenica* nov. sp., belongs to the subfamily *Aproctinae* Yorke et Maplestone, 1926. The most closely related to it are: *Pelecitus tercostatus* Mol., 1859, and *Pelecitus helix* Lins., 1899. From those two species the new species under description differs however 1) by the number of its cephalic suckers which are 10 in *P. tercostatus*, 6 in *Pelecitus helix*, and 14 in *P. armenica* nov. sp.; 2) according to the size of spicules (in *P. tercostatus* the length of spicules attains 0.136 mm against a width of 0.085 mm, in *P. helix* the respective figures are 0.11 and 0.077, whereas in *P. armenica* nov. sp. they are 0.150 and 0.110); 3) according to the localization of the anus in females: in females of *P. tercostatus* the anus is situated at a distance of 0.085 mm from the caudal body end; in *P. helix* the respective figure is 0.09, whereas in *P. armenica* nov. sp. the anus is situated terminally, or nearly so.

It is necessary to emphasize here the great difficulties involved in a differential diagnostication of certain *Pelecitus* species, which are caused by the fact that the 5 species belonging to this genus have been but poorly investigated (*P. helicinus* Mol., 1860; *P. circularis* Mol., 1860; *P. quadripapillosus* Mol., 1859; *P. calamiformis* Schn., 1866; *P. serpentulus* Dies., 1851).

In concluding it is a pleasure to express my sincere thanks to K. I. Skrjabin, Member of the Academy, for having suggested the theme of this investigation, and to Professor A. M. Petrov, for the continuous interest he showed in the course of the work.

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